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Convex Optimization

Winter Term 2015/16

— Exercise Sheet 4 —

Exercise 4.1. Draw the cone of positive semidefinite matrices

$$\mathcal{S}^2_{\succeq 0} = \left\{ X \in \mathcal{S}^2 : X = \begin{pmatrix} X_{11} & X_{12} \\ X_{12} & X_{22} \end{pmatrix} \text{ is positive semidefinite} \right\}.$$

Exercise 4.2. Determine the extreme points of the compact convex set

$$\mathcal{S}^n_{\succeq 0} \cap \{X \in \mathcal{S}^n : \mathrm{Tr}(X) = 1\}.$$

Exercise 4.3. Consider the semidefinite optimization problem

$$\begin{aligned} \text{maximize } & X_{11} + 3X_{12} \\ \text{subject to } & 2X_{11} + X_{22} = 1, \\ & \begin{pmatrix} X_{11} & X_{12} \\ X_{12} & X_{22} \end{pmatrix} \succeq 0. \end{aligned}$$

1. Draw the set of feasible solutions.
2. Determine the optimal solutions.
3. Determine the dual program and its optimal solutions.

Exercise 4.4 Let $K \subseteq \mathbb{R}^n$ be a proper convex cone and let $A \in \mathbb{R}^{m \times n}$ be a matrix and $c \in \mathbb{R}^n$ be a vector. Show that exactly one of the following two alternatives holds:

1. There exists $x \in K$ with $Ax = 0$ and $c^{\mathsf{T}} x > 0$.
2. The system $A^{\mathsf{T}} y - c \in K^*$ is weak feasible, i.e. for all $\varepsilon > 0$ there exists a vector $y \in \mathbb{R}^m$ and an element $z \in K^*$ such that $\|A^{\mathsf{T}} y - c - z\| \leq \varepsilon$.

Hand-in: Until Tuesday, 17th November, 2pm at the “Convex optimization” mailbox in room 3.01 (Studierendenarbeitsraum) of the Mathematical Institute. Please add your name, student number, and group number to your solution sheet.