



Universität zu Köln
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Convex Optimization

Winter Term 2015/16

— Exercise Sheet 5 —

Exercise 5.1. Let $K \subseteq \mathbb{R}^n$ be a proper convex cone, let $A \in \mathbb{R}^{m \times n}$ be a matrix and $c \in \mathbb{R}^n$ be a vector. Show that exactly one of the following two alternatives holds:

1. There exists $x \in K \setminus \{0\}$ with $Ax = 0$ and $c^\top x \geq 0$.
2. There exists $y \in \mathbb{R}^m$ such that $A^\top y - c \in \text{int } K^*$.

Exercise 5.2. Let $\hat{r} = (3, 2.65, 2, 1.9, -1.5)$ be a vector of effective dividend returns, i.e. the fraction of returns given by dividend payments of a stock divided by the capital held in this stock, reduced by the inflation rate. Let $B(\hat{r}, 1) = \{z : \|z - \hat{r}\|_2 \leq 1\}$ be the unit ball around \hat{r} . Determine the value of the optimal dividend strategy given by the following program:

$$\max_{w \in \mathbb{R}^5} \min_{r \in B(\hat{r}, 1)} \{r^\top w : w^\top e = 1, w \geq 0\},$$

where $e = (1, 1, 1, 1, 1)^\top$.

Exercise 5.3. Let $C \in \mathcal{S}^n$ and $k \in \mathbb{N}$ be given. Consider the following maximization problem

$$\max_{X \in \mathcal{S}^n} \{ \langle C, X \rangle : \text{Tr}(X) = k, I_n \succeq X \succeq 0 \}.$$

1. Write the given problem as a semidefinite program.
2. Determine the corresponding dual program.

Exercise 5.4. What is the exact value of the minimal maximal eigenvalue of the following matrix $X \in \mathcal{S}^5$

$$\begin{pmatrix} 1 & X_{12} & 1 & X_{14} & X_{15} \\ X_{12} & 1 & 1 & X_{24} & X_{25} \\ 1 & 1 & 1 & X_{34} & 1 \\ X_{14} & X_{24} & X_{34} & 1 & 1 \\ X_{15} & X_{25} & 1 & 1 & 1 \end{pmatrix} ?$$

Hint: You can use a numerical SDP solver to “guess” the solution.

Hand-in: Until Tuesday, 24th November, 2pm at the “Convex optimization” mailbox in room 3.01 (Studierendenarbeitsraum) of the Mathematical Institute. Please add your name, student number, and group number to your solution sheet.