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Mathematisches Institut
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Convex Optimization

Winter Term 2015/16

— Exercise Sheet 6 —

Exercise 6.1. A polyomial $p \in \mathbb{R}[x]$ is said to be a *sum of squares (SOS)*, if there exist polynomials $q_1, \ldots, q_m$, such that $p = \sum_{i=1}^m q_i^2$. Show that a univariate polynomial p of degree at most $2d$ is SOS if and only if there exists a positive semidefinite matrix $Q \in \mathcal{S}_{\succeq 0}^{d+1}$ such that

$$p(x) = (1, x, x^2, \ldots, x^d)\, Q\, (1, x, x^2, \ldots, x^d)^{\mathsf{T}}.$$

Exercise 6.2. Is the following polynomial a SOS

$$x_1^6 + x_2^6 + x_3^6 - (x_1^4x_2^2 + x_1^2x_2^4 + x_1^4x_3^2 + x_1^2x_3^4 + x_2^4x_3^2 + x_2^2x_3^4) + 3x_1^2x_2^2x_3^2 \ ?$$

Exercise 6.3. Let $P_1, P_2 \ldots, P_n$ be finite subsets having union $M = \bigcup_{i=1}^n P_i$. The problem SET COVER is finding a selection $I \subseteq \{1, \ldots, n\}$ of smallest cardinality so that $M = \bigcup_{i \in I} P_i$ holds. Determine an SDP relaxation for SET COVER.

Exercise 6.4. A *singular value decomposition* of a matrix $X \in \mathbb{R}^{m \times n}$ is of the following form:

$$X = UYV^{\mathsf{T}},$$

where $Y \in \mathbb{R}^{m \times n}$ is a diagonal matrix with non-negative diagonal entries and $U \in \mathrm{O}(m)$ and $V \in \mathrm{O}(n)$ are orthogonal matrices. The diagonal entries of Y are called *singular values* of X.

1. Show that the singular values of X are equal to the nonnegative eigenvalues of the following matrix
$$\begin{pmatrix} 0 & X^{\mathsf{T}} \\ X & 0 \end{pmatrix}.$$
2. Determine the minimal maximal singular value of the following matrix
$$\begin{pmatrix} 1 & x_1 & 1 \\ 5 & 1 & x_2 \end{pmatrix}.$$

Hand-in: Until Tuesday, 1st December, 2pm at the "Convex optimization" mailbox in room 3.01 (Studierendenarbeitsraum) of the Mathematical Institute. Please add your name, student number, and group number to your solution sheet.

Hey Students!
The Math-Student Association is celebrating Christmas! You are invited to drink a hot mulled wine in the Asta Café or eat a warm waffle on December 9th at 7 pm. We are looking forward to seeing you!