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Convex Optimization

Winter Term 2015/16

— Exercise Sheet 9 —

Exercise 9.1. Use Davis' characterization of convex and spectral functions to show: The function

$$F : \mathcal{S}^n \to \mathbb{R} \cup \{\infty\}, \quad F(X) = \begin{cases} -\ln \det X, & \text{if } X \succ 0, \\ \infty, & \text{otherwise,} \end{cases}$$

is convex and spectral.

Exercise 9.2. Let $X, Y \in \mathcal{S}^n$ be symmetric matrices. Determine the minimum

$$\min_{A \in \mathsf{O}(n)} \langle X, AYA^{\mathsf{T}} \rangle.$$

Exercise 9.3. Let $P = \{x \in \mathbb{R}^n : a_j^{\mathsf{T}} x \leq b_j, j \in [m]\}$ be an n-dimensional polytope. Formulate the following problem as a conic program: Find the largest volume of an axis-parallel parallelepiped

$$R = \{x \in \mathbb{R}^n : \alpha_1 \leq x_1 \leq \beta_1, \ldots, \alpha_n \leq x_n \leq \beta_n\},$$

with $R \subseteq P$.

Exercise 9.4. Let $C \in \mathcal{S}^n$ be a symmetric matrix and let $G = (V, E)$ be a graph with vertex set $V = [n]$. The solution of the following MAXDET problem

$$\begin{aligned} \max \quad & \det\left(C + \sum_{\{i,j\} \in E} x_{ij} E_{ij}\right)^{1/n} \\ & C + \sum_{\{i,j\} \in E} x_{ij} E_{ij} \in \mathcal{S}^n_{\succeq 0}, \end{aligned}$$

is said to be a *G-modification of C with maximal entropy*. Show: If a G-modification of C with maximal entropy $A^* = C + \sum_{\{i,j\} \in E} x_{ij}^* E_{ij}$ exists, then

$$\forall \{i,j\} \in E \ : \ ((A^*)^{-1})_{ij} = 0.$$

Hand-in: Until Tuesday, 22nd December, 2pm at the "Convex optimization" mailbox in room 3.01 (Studierendenarbeitsraum) of the Mathematical Institute. Please add your name, student number, and group number to your solution sheet.