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Convex Optimization

Winter Term 2015/16

— Exercise Sheet 10 —

Exercise 10.1. Let $P \subseteq \mathbb{R}^n$ be a centrally symmetric polytope ($P = -P$). Find a conic program (with possibly infinitely many constraints) which determines the minimal value $\rho \in \mathbb{R}$ such that there exists an ellipsoid E for which $E \subseteq P \subseteq \rho E$ holds.

Exercise 10.2. Let $P = \{y \in \mathbb{R}^n : a_1^{\mathsf{T}} y \leq b_1, \ldots, a_m^{\mathsf{T}} y \leq b_m\}$ be a polytope. Consider the inner ellipsoid $\mathcal{E}(A^2, x) \subseteq P$ with maximal volume, which is determined by the optimal solution of the following maximization problem

$$\begin{aligned} &\max s \\ &\qquad (A, s) \in \mathcal{D}^n, x \in \mathbb{R}^n, \\ &\qquad (Aa_j, b_j - a_j^T x) \in \mathcal{L}^{n+1}, \text{ for } j \in [m]. \end{aligned}$$

Determine the corresponding dual program and prove that strong duality holds.

Exercise 10.3. Let $P \subseteq \mathbb{R}^n$ be a full dimensional polytope. Apply polarity to show that the fact $\mathcal{E}_{in}(P) = B_n$ is equivalent to the fact that $B_n \subseteq P$ and that there are positive real numbers $\lambda_1, \ldots, \lambda_M$ and vectors $x_1, \ldots, x_M \in \partial P$ so that the following three conditions are fulfilled:

1. $\|x_i\| = 1$, for $i = 1, \ldots, M$,
2. $\sum_{i=1}^M \lambda_i x_i = 0$,
3. $\sum_{i=1}^M \lambda_i x_i x_i^{\mathsf{T}} = I_n$.

Exercise 10.4. Determine the Löwner-John ellipsoid $\mathcal{E}_{\text{in}}(C_n)$ of the regular n-gon C_n in the plane

$$C_n = \text{conv}\{(\cos(2\pi k/n), \sin(2\pi k/n)) \in \mathbb{R}^2 : k = 0, 1, \ldots, n-1\}.$$

Hand-in: Until Tuesday, 12th January, 2pm at the “Convex optimization” mailbox in room 3.01 (Studierendenarbeitsraum) of the Mathematical Institute. Please add your name, student number, and group number to your solution sheet.

* * * *Merry Christmas and a Happy New Year !* * * *