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Convex Optimization

Winter Term 2015/16

— Exercise Sheet 12 —

Exercise 12.1. Let $G = (V, E)$ be a graph. Show:

$$\vartheta(G) = \min\{t : K \in \mathcal{S}^V, K - J \in \mathcal{S}^V_{\succeq 0}, K_{ii} \leq t \text{ for } i \in V, K_{ij} = 0 \text{ if } \{i,j\} \in \overline{E}\}.$$

Exercise 12.2. Let $G = (V, E)$ be a graph and let X be an optimal solution of the semidefinite program computing $\vartheta(G)$ (Definition VII.2.1). Prove:

$$Xe = \vartheta(G) \operatorname{diag} X,$$

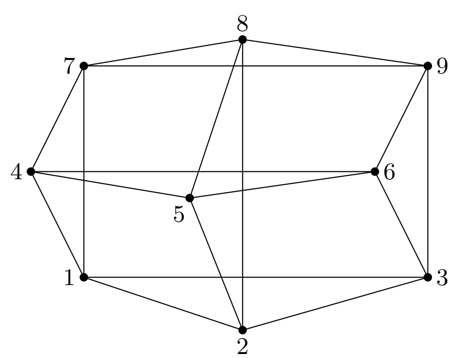
where $e = (1, \ldots, 1)^{\mathsf{T}}$ and diag X is the vector containing the diagonal entries of X.

Exercise 12.3. Let $G = (V, E)$ be a bipartite graph and let $L(G)$ be the line graph of G defined by

$$L(G) = (E, \{\{e, f\} : |e \cap f| = 1\}).$$

Show: $L(G)$ is perfect.

Exercise 12.4. Show that the graph G on the right is a perfect graph. Compute an independent set I in G with $|I| = \alpha(G)$ using the algorithm given in the lecture.



Hand-in: Until Tuesday, 26th January, 2pm at the "Convex optimization" mailbox in room 3.01 (Studierendenarbeitsraum) of the Mathematical Institute. Please add your name, student number, and group number to your solution sheet.