3) $A_2(11,6) \leq 12$

Mittels Delsarte LP-Schranke.

Verstärke die LP-Schranke durch

a) $\varphi_c(x-y) \geq 0$, falls $w(x-y) \in \{6,8,10\}$

($\rightsquigarrow$ Variablen $\beta_6, \beta_8, \beta_{10} \leq 0$ im Dualen)

b) $\varphi_c(x-y) = 0$, falls $w(x-y) \in \{7,9,11\}$.

```
Q := RationalField();
P<x,q,n> := PolynomialRing(Q,3);

function binom(p,j)
  out := P!1;
  for i := 0 to j-1 do
    out := out*(p-i);
  end for;
  return out/Factorial(j);
end function;

Krawtchouk := [];

m := 11;

for k := 0 to m do
  K := P!0;
  for j := 0 to k do
    K := K + (-1)^j * binom(x,j) * binom(n-x,k-j) * (q-1)^(k-j);
  end for;
  Append(~Krawtchouk,K);
end for;

function K(k,i,q,n)
  return Evaluate(Krawtchouk[k+1],[i,q,n]);
end function;


n := 11;
d := 6;

printf "min: b0;\n";

printf "r%o: b0 ", 0;
for k in [1..n] do
  if k in [6,8,10] then
    printf "- %o b%o ", Binomial(n,k), k;
  else
    printf "+ %o b%o ", Binomial(n,k), k;
  end if;
end for;
printf ">= %o;\n", 2^n;

for i := 1 to n do
  printf "r%o: b0 ", i;
  for k in [1..n] do
    val := K(k,i,2,n);
    if k in [6,8,10] then
      val := -val;
    end if;
    if val ge 0 then
      printf "+";
    end if;
    printf "%o b%o ", val, k;
  end for;
  printf ">= 0;\n";
end for;
```

```
min: b0;
r0: b0 + 11 b1 + 55 b2 + 165 b3 + 330 b4 + 462 b5 - 462 b6 + 330 b7 -
165 b8 + 55 b9 - 11 b10 + 1 b11 >= 2048;
r1: b0 +9 b1 +35 b2 +75 b3 +90 b4 +42 b5 +42 b6 -90 b7 +75 b8 -35 b9
+9 b10 -1 b11 >= 0;
r2: b0 +7 b1 +19 b2 +21 b3 -6 b4 -42 b5 +42 b6 -6 b7 -21 b8 +19 b9 -7
b10 +1 b11 >= 0;
r3: b0 +5 b1 +7 b2 -5 b3 -22 b4 -14 b5 -14 b6 +22 b7 -5 b8 -7 b9 +5
b10 -1 b11 >= 0;
r4: b0 +3 b1 -1 b2 -11 b3 -6 b4 +14 b5 -14 b6 -6 b7 +11 b8 -1 b9 -3
b10 +1 b11 >= 0;
r5: b0 +1 b1 -5 b2 -5 b3 +10 b4 +10 b5 +10 b6 -10 b7 -5 b8 +5 b9 +1
b10 -1 b11 >= 0;
r6: b0 -1 b1 -5 b2 +5 b3 +10 b4 -10 b5 +10 b6 +10 b7 -5 b8 -5 b9 +1
b10 +1 b11 >= 0;
r7: b0 -3 b1 -1 b2 +11 b3 -6 b4 -14 b5 -14 b6 +6 b7 +11 b8 +1 b9 -3
b10 -1 b11 >= 0;
r8: b0 -5 b1 +7 b2 +5 b3 -22 b4 +14 b5 -14 b6 -22 b7 -5 b8 +7 b9 +5
b10 +1 b11 >= 0;
r9: b0 -7 b1 +19 b2 -21 b3 -6 b4 +42 b5 +42 b6 +6 b7 -21 b8 -19 b9 -7
b10 -1 b11 >= 0;
r10: b0 -9 b1 +35 b2 -75 b3 +90 b4 -42 b5 +42 b6 +90 b7 +75 b8 +35 b9
+9 b10 +1 b11 >= 0;
r11: b0 -11 b1 +55 b2 -165 b3 +330 b4 -462 b5 -462 b6 -330 b7 -165 b8
-55 b9 -11 b10 -1 b11 >= 0;

Value of objective function: 12.00000000

Actual values of the variables:
b0                              12
b1                            1.44
b2                             6.4
b3                               0
b4                             2.4
b5                            0.16
b6                               0
b7                            1.92
b8                             0.8
b9                            5.28
b10                              0
b11                          10.24
```

Alternativer Beweis der Plotkin-Schranke

Plotkin: $\Theta = 1 - \frac{1}{q}$, $d > \Theta n$, $A_q(n,d) \leq \frac{d}{d - \Theta n}$.

Bew.: Verwende alternativen (aber äquivalenten) Zugang zur LP-Schranke.

Sei $C \subseteq \mathbb{F}_q^n$ ein (n, M, d)-Code. Definiere die **Distanzverteilung**

$$A_i = \frac{1}{|C|} \left| \{ (x,y) \in C \times C : w(x-y) = i \} \right|.$$

Dann gilt:

a) $A_0 = 1$

b) $A_1 = \ldots = A_{d-1} = 0$

c) $A_d, \ldots, A_n \geq 0$

d) $\sum_{i=0}^{n} A_i = |C|$

e) $\sum_{i=0}^{n} A_i K_k(i) \geq 0, \quad k = 1, \ldots, n.$

a) – d): klar.

zu e):

$$|C| \sum_{i=0}^{n} A_i K_k(i) = \sum_{i=0}^{n} \sum_{\substack{(x,y)\in C\times C \\ w(x-y)=i}} K_k(i)$$

$$= \sum_{i=0}^{n} \sum_{\substack{(x,y) \\ w(x-y)=i}} \sum_{\substack{z \\ w(x)=k}} \chi_{x-y}(z)$$

$$= \sum_{\substack{z \\ w(x)=k}} \sum_{(x,y)\in C} \chi_z(x) \overline{\chi_z(y)}$$

$$= \sum_{z} \left| \sum_{x\in C} \chi_z(x) \right|^2 \geq 0.$$

Also:

$$A_q(n,d) \leq \max \sum_{i=0}^{n} x_i$$

$$x_0 = 1,\ x_1 = \ldots = x_{d-1} = 0,\ x_d, \ldots, x_n \geq 0$$

$$\sum_{i=0}^{n} K_k(i)\, x_i \geq 0, \quad k = 1, \ldots, n.$$

Nun zur Plotkin-Schranke: Verwende nur die Bedingung $k=1$.

Es ist $K_1(i) = -qi + n(q-1)$.

Dann

$$A_q(n,d) \leq \max \; 1 + x_d + \dots + x_n$$

$$x_d \geq 0, \dots, x_n \geq 0$$

$$n(q-1) + [-qd + n(q-1)]x_d +$$

$$+ [-q(d+1) + n(q-1)]x_{d+1} + \dots + [-qn + n(q-1)]x_n$$

$$\geq 0$$

Damit $1 + x_d + \dots + x_n$ möglichst groß ist, sollte x_d möglichst groß sein, da $-qi + n(q-1) < 0$, $i = d, \dots, n$ ist. (weil $d > \theta n$).

Also $x_{d+1} = \dots = x_n = 0$ und

$$n(q-1) + (-qd + n(q-1))x_d = 0$$

$$\Longleftrightarrow \quad x_d = \frac{n(1-q)}{n(q-1) - qd}$$

Und somit

$$A_q(n,d) \leq 1 + \frac{n(q-1)}{qd - n(q-1)} = \frac{qd}{qd - n(q-1)} = \frac{d}{d - \theta n}.$$

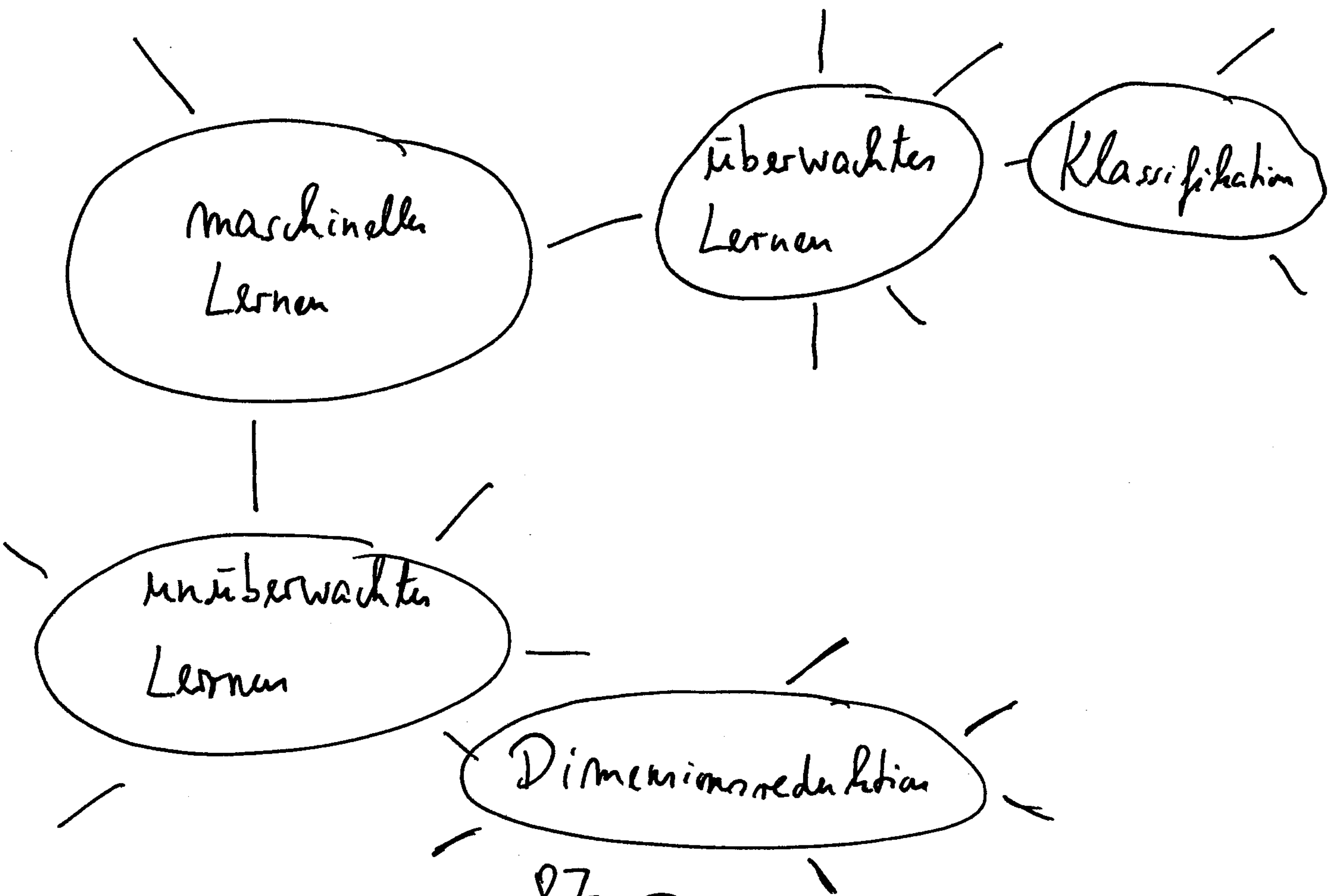
$\boxtimes$

Teil B Compressive Sensing

Aus Zeitgründen: $\rightarrow$ Seminar nächstes Semester

Teil C Maschinelles Lernen

Diagramm in Barber – Bayesian Reasoning and Machine Learning:



Bsp.: überwachtes Lernen

geg.: $\mathcal{D} = \{(x_n, y_n) : n = 1, \dots, 10000\}$ Datenbank von Bildern mit Gesichtern x_n mit Information $y_n \in \{m, w\}$.

Ziel: Für ein neues x^* finde eine möglichst präzise Voraussage $y^* \in \{m, w\}$.

Bsp.: unüberwachtes Lernen

geg.: $\mathcal{D} = \{x_n \in \{0,1\}^{10000} : n = 1, \dots, 10^7\}$

	x_1	x_2	x_3	x_4	…
Kaffee	1	0	0	0	
Tee	0	0	1	0	
Milch	1	0	1	0	
Bier	0	0	0	1	
Windeln	0	0	1	1	
Aspirin	0	1	0	1	

Ziel: Finde Muster in den Käufen.