Universität zu Köln
Mathematisches Institut
Prof. Dr. F. Vallentin
Dr. A. Gundert
Dr. F. von Heymann

Convex Optimization

Winter Term 2018/19

— Exercise Sheet 6 —

Exercise 6.1. Let $X \in \{\pm 1\}^{n\times n}$ be a symmetric matrix whose entries are 1 or -1. Show that X is positive semidefinite if and only if $X = xx^{\mathsf{T}}$ for some $x \in \{\pm 1\}^n$.

Exercise 6.2. Let $G = (V, E)$ be a graph and $w \in \mathbb{R}^E_+$ a nonnegative weight function on the edges of G. Show:

$$\operatorname{sdp}(G, w) \leq \sum_{\{i,j\}\in E} w_{ij}.$$

What does this imply for bipartite graphs?

Exercise 6.3. (Hand-in)

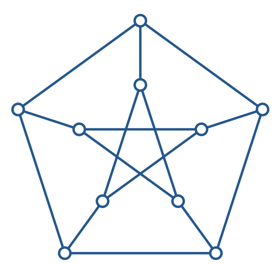
(a) Let $G = (V, E)$ be a graph with $|V| = n$ and non-negative weigths $w \in \mathbb{R}^{|E|}_+$. Show:

$$\begin{aligned} \operatorname{mc}(G, w) = \max \; & \sum_{\{i,j\}\in E} w_{ij} \frac{\arccos(v_i^{\mathsf{T}} v_j)}{\pi} \\ & v_1, \ldots, v_n \in \mathbb{R}^n \\ & \|v_i\| = 1 \text{ for all } i \in [n] \end{aligned}$$

(b) Show: For unit-vectors $v_1, \ldots, v_7$ we have:

$$\sum_{1\leq i<j\leq 7} \arccos(v_i^{\mathsf{T}} v_j) \leq 12\pi.$$

Exercise 6.4. (Hand-in)



(a) Use a computer to solve $\operatorname{sdp}(P, \mathbf{1})$ for the Peterson graph P (see figure), then use your result to determine $\operatorname{mc}(P, \mathbf{1})$. Here $\mathbf{1}$ denotes the constant weight function with $w_i = 1$, for $i \in V$.

(b) Let $C_n = (V_n, E_n)$ be the cycle graph with n vertices, defined by

$$V_n = \{1, \ldots, n\} \quad \text{and} \quad E_n = \{\{1, 2\}, \{2, 3\}, \ldots, \{n-1, n\}, \{n, 1\}\}.$$

Compute $\operatorname{sdp}(C_n, \mathbf{1})$ for $n = 2k + 1$ and $k \in \{1, 2, 3, 4\}$.

Hand-in: Until Wednesday November 21, 12:00 (noon).
Exercises 6.3 and 6.4 to be submitted to the “Convex optimization” mailbox in room 3.01 (Studierendenarbeitsraum) of the Mathematical Institute.