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Convex Optimization

Winter Term 2018/19

— Exercise Sheet 7 —

Exercise 7.1. Let $G = (V, E)$ be a graph. Determine the dual of the following semidefinite program and prove that strong duality holds

$$\begin{aligned} \vartheta^+(G) = \max \quad & \langle J, X \rangle \\ & X \succeq 0 \\ & \mathrm{Tr}(X) = 1 \\ & X_{ij} \leq 0 \text{ for } \{i, j\} \in E. \end{aligned}$$

Exercise 7.2. Let $G = (V, E)$ be a graph. Find a formulation of $\alpha(G)$ as a quadratic program (see Definition V.1.1) such that the semidefinite relaxation of this program (see Theorem V.1.2) is equal to $\vartheta(G)$.

Exercise 7.3. (Hand-in) Let $G = (V, E)$ be a graph and let X be an optimal solution of the semidefinite program computing $\vartheta(G)$. Prove:

$$Xe = \vartheta(G) \operatorname{diag} X,$$

where $e = (1, \ldots, 1)^{\mathsf{T}}$ and diag X is the vector containing the diagonal entries of X.

Hint: For any $X, Y \in \mathcal{S}^n_+$ we have $\langle X, Y \rangle = 0$ if and only if $XY = 0$. (Why?)

Exercise 7.4. (Hand-in)

(a) Compute $\alpha(P)$, $\omega(P)$, $\chi(P)$, and $\vartheta(P)$ for the Peterson graph P (see the figure on exercise sheet 6).

(b) Let $C_n = (V_n, E_n)$ be the cycle graph with n vertices, see Exercise 6.4. Compute $\vartheta(C_n)$ for $n = 2k + 1$ and $k \in \{1, 2, 3, 4\}$.

Hand-in: Until Wednesday November 28, 12:00 (noon).
Exercises 7.3 and 7.4 to be submitted to the "Convex optimization" mailbox in room 3.01 (Studierendenarbeitsraum) of the Mathematical Institute.