Universität zu Köln
Mathematisches Institut
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Convex Optimization

Winter Term 2018/19

— Exercise Sheet 9 —

Exercise 9.1.

(a) Find a $(\mathbb{Z}/6\mathbb{Z})$-circulant matrix with the following 6 eigenvalues: $\{-12, 0, 2, 3, 12, 18\}$.

(b) Compute the determinant of the following matrix:

$$\begin{pmatrix}
-6 & 2 & 12 & -10 & 4 & 6 & 3 & -2 & 4 & 6 & 12 & -10 \\
2 & -6 & -10 & 12 & 6 & 4 & -2 & 3 & 6 & 4 & -10 & 12 \\
12 & -10 & -6 & 2 & 12 & -10 & 4 & 6 & 3 & -2 & 4 & 6 \\
-10 & 12 & 2 & -6 & -10 & 12 & 6 & 4 & -2 & 3 & 6 & 4 \\
4 & 6 & 12 & -10 & -6 & 2 & 12 & -10 & 4 & 6 & 3 & -2 \\
6 & 4 & -10 & 12 & 2 & -6 & -10 & 12 & 6 & 4 & -2 & 3 \\
3 & -2 & 4 & 6 & 12 & -10 & -6 & 2 & 12 & -10 & 4 & 6 \\
-2 & 3 & 6 & 4 & -10 & 12 & 2 & -6 & -10 & 12 & 6 & 4 \\
4 & 6 & 3 & -2 & 4 & 6 & 12 & -10 & -6 & 2 & 12 & -10 \\
6 & 4 & -2 & 3 & 6 & 4 & -10 & 12 & 2 & -6 & -10 & 12 \\
12 & -10 & 4 & 6 & 3 & -2 & 4 & 6 & 12 & -10 & -6 & 2 \\
-10 & 12 & 6 & 4 & -2 & 3 & 6 & 4 & -10 & 12 & 2 & -6
\end{pmatrix}.$$

Exercise 9.2. Let G be a finite Abelian group. Show:

(a) G-circulant matrices commute.

(b) If a G-circulant matrix is invertible, the inverse is G-circulant again.

Exercise 9.3. (Hand-in) Formulate $\vartheta(\text{Cayley}(\mathbb{Z}/11\mathbb{Z}, \{1, -1, 5, -5\}))$ as a linear program and use a computer to solve this.

Exercise 9.4. (Hand-in) Let G be a finite Abelian group and $x \in G$.

(a) Derive a linear program with two equality constraints that computes $\vartheta(\text{Cayley}(G, \{x, -x\}))$.

(b) Dualize the linear program from (a).

(c) Find a formula for $\vartheta(\text{Cayley}(G, \{x, -x\}))$ by solving the program from (b).

Hand-in: Until Wednesday December 12, 12:00 (noon).
Exercises 9.3 and 9.4 to be submitted to the "Convex optimization" mailbox in room 3.01 (Studierendenarbeitsraum) of the Mathematical Institute.