Universität zu Köln
Mathematisches Institut
Prof. Dr. F. Vallentin
Dr. A. Gundert
Dr. F. von Heymann

Convex Optimization

Winter Term 2018/19

— Exercise Sheet 10 —

Exercise 10.1. Determine the values f_k, with $k = 0, \ldots, 6$, satisfying

$$\sum_{k=0}^{6} f_k P_k^8(t) = -1 + \frac{320}{3}(t+1)(t+1/2)^2 t^2 (t-1/2)$$

and finish the proof that $\tau_8 = 240$.

Exercise 10.2. Let $C \subseteq S^{n-1}$ be a finite subset. For $n \geq 3$, define the Newton potential energy of C by

$$\sum_{x,y \in C, x \neq y} \frac{1}{\|x-y\|^{n-2}}.$$

Let F be a polynomial of the form

$$F(t) = \sum_{k=0}^{d} f_k P_k^n(t) \quad \text{with} \quad f_0, \ldots, f_d \geq 0.$$

Suppose $F(t) \leq 1/(2-2t)^{\frac{n-2}{2}}$ for all $t \in [-1, +1]$. Show: Every set of N points $C \subseteq S^{n-1}$ has Newton potential energy at least $N^2 f_0 - NF(1)$.

Exercise 10.3. (Hand-in) Show that $\alpha(G(n, \pi/2)) = 2n$.

Exercise 10.4. (Hand-in) Write a computer program to determine (an approximation of) $\vartheta'(G(n, \pi/3))$ for $n = 2, 3, \ldots, 24$.

Hand-in: Until Wednesday December 19, 12:00 (noon).
Exercises 10.3 and 10.4 to be submitted to the "Convex optimization" mailbox in room 3.01 (Studierendenarbeitsraum) of the Mathematical Institute.