Universität zu Köln
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Convex Optimization

Winter Term 2018/19

— Exercise Sheet 11 —

Exercise 11.1. For which $k \in \mathbb{N}$ is the function

$$\Phi_k : \mathcal{S}^n \to \mathbb{R},\ X \mapsto \text{Tr}(X^k)$$

a convex spectral function?

Exercise 11.2. Let $P = \{x \in \mathbb{R}^n : a_j^\mathsf{T} x \leq b_j, j \in [m]\}$ be an n-dimensional polytope. Formulate the following problem as a conic program: Find the largest volume of an axis-parallel parallelepiped

$$R = \{x \in \mathbb{R}^n : \alpha_1 \leq x_1 \leq \beta_1, \ldots, \alpha_n \leq x_n \leq \beta_n\},$$

with $R \subseteq P$.

Exercise 11.3. (Hand-in) Use Davis' characterization of convex and spectral functions to show: The function

$$F : \mathcal{S}^n \to \mathbb{R} \cup \{\infty\}, \quad F(X) = \begin{cases} -\ln \det X, & \text{if } X \succ 0, \\ \infty, & \text{otherwise,} \end{cases}$$

is convex and spectral.

Exercise 11.4. (Hand-in) Let $X, Y \in \mathcal{S}^n$ be symmetric matrices. Determine the minimum

$$\min_{A \in \mathsf{O}(n)} \langle X, AYA^\mathsf{T} \rangle.$$

Hand-in: Until Wednesday January 16, 12:00 (noon).
Exercises 11.3 and 11.4 to be submitted to the "Convex optimization" mailbox in room 3.01 (Studierendenarbeitsraum) of the Mathematical Institute.