New lower bounds on crossing numbers of $K_{m,n}$ from permutation modules and semidefinite programming

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Abstract

In this talk, we use semidefinite programming and representation theory to compute new lower bounds on the crossing number of the complete bipartite graph $K_{m,n}$, extending a method from de Klerk et al. [SIAM J. Discrete Math. 20 (2006), 189–202] and the subsequent reduction by De Klerk, Pasechnik and Schrijver [Math. Prog. Ser. A and B, 109 (2007) 613–624].

We exploit the full symmetry of the problem by developing a block-diagonalization of the underlying matrix algebra and use it to improve bounds on several concrete instances. Our results imply that $\mathrm{cr}(K_{10,n}) \geq 4.87057n^2 - 10n$, $\mathrm{cr}(K_{11,n}) \geq 5.99939n^2 - 12.5n$, $\mathrm{cr}(K_{12,n}) \geq 7.25579n^2 - 15n$, $\mathrm{cr}(K_{13,n}) \geq 8.65675n^2 - 18n$ for all n. The latter three bounds are computed using a new relaxation of the original semidefinite programming bound, by only requiring one small matrix block to be positive semidefinite. Our lower bound on $K_{13,n}$ implies that for each fixed $m \geq 13$, $\lim_{n\to\infty} \mathrm{cr}(K_{m,n})/Z(m,n) \geq 0.8878m/(m-1)$. Here $Z(m,n)$ is the *Zarankiewicz number*: the conjectured crossing number of $K_{m,n}$.

This talk is based on joint work with Sven Polak.