Evaluating the connectivity of a graph using semidefinite programming

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Abstract

There are several ways to measure the connectivity of a graph. The most famous one is the Fiedler value, which is the second smallest eigenvalue of the Laplacian matrix. Another way to express the connectivity of a graph is the so-called edge expansion. This graph constant is the minimum ratio of the number of edges joining two sets and the size of the smaller set over all possible non-trivial bipartitions of the vertices. The edge expansion of a connected graph is small if there is a bottleneck between two large parts of the graph. Because of this fact, it is for example used in clustering or network design. Unfortunately, this graph constant is NP-hard to compute.

The fractional optimization problem to compute the edge expansion does not fit into the typical setting like other well-known graph constants as the maximum cut which are NP-hard to compute. We propose different strategies to compute the edge expansion efficiently. One is to divide the problem into subproblems of an easier-to-handle type. Another one is to apply Dinkelbach's algorithm for fractional programming. Furthermore, we investigate the conjecture of Mihail and Vazirani, stating that the edge epxansion of the graph from a 0/1-polytope is at least 1, using our techniques.