Near toric integrable systems

Abstract: In recent years there has been interest in studying families of integrable systems that generalise integrable toric actions while retaining important similarities, like connectedness of the fibres. One such family is that of semitoric systems on four dimensional compact symplectic manifolds: these are integrable systems one of whose integrals generates a circle action and with the property that all singular orbits are either of the types that occur in toric manifolds or are complex hyperbolic (focus-focus). These systems have been intensely studied in recent years: Pelayo and Vu Ngoc have classified them and Le Floch, Pelayo and Vu Ngoc have studied their quantum counterparts. However, there are reasons to look beyond semitoric systems. First, it can be shown that not all compact four dimensional symplectic manifolds endowed with a Hamiltonian circle action possess one such system. Second, the analogous definition in higher dimensions is rather restrictive and does not allow for singularities that appear naturally, say, when considering special Lagrangian fibrations. In this talk we introduce the class of near toric systems that generalises semitoric systems in any dimensions and (should!) allow to deal with the above issues. The aim of the talk is to prove that the fibres of such systems on compact symplectic manifolds are connected and, time permitting, to illustrate how one may go about trying to classify them. This is based on joint work with Susan Tolman.