Bibliographic references for the course on

Action minimizing periodic solutions of the N-body problem

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Only references the most directly related to the contents of the course are cited below.

1. The N-body problem and its symmetries

The book [4] is an invaluable reference for the N-body problem. The article [13] is a short, introductory version to the topic.

The other book of Arnold [3] is less specialized and provides an excellent, conceptual introduction to the mathematical methods in classical mechanics, with a lot of geometric insight. The appendices are particularly remarkable.

The article of Albouy-Chenciner [2] focuses on the symmetries of the N-body problem in $\mathbb{R}^d$, the corresponding reductions (generalizing the method of Lagrange for the 3-bordy problem), and homographic solutions.

2. Homographic solutions

Again, we refer to the book [4], the article [2] and references therein.

3. The Lagrangian action and its minima

The PhD thesis of Venturelli [19] (in particular the introduction, the appendices, as well of the first chapter) and the book of Young [20] provide us with some details about the "well-known" facts about the Lagrangian action functional: weak lower semi-continuity, differentiability, Gordon's theorem, etc. See also [7].

The theorem of Marchal, which has become a corner point in the subject, is described somewhat informally in [16] and with more details in Chenciner's ICM address [7]; a more general, equivariant form is in the article of Ferrario-Terracini [14].

4. The eight-shaped solution of the equal-mass three body problem

The article of Chenciner-Montgomery [11] gives the first proof of existence of an action minimizing periodic orbit using a direct method. Chen [5] has improved the part of the proof which shows that the minimizer is collision free.

5. The P_{12} family

This family relating the Eight to the Lagrange relative equilibrium is studied in various articles : the starting (wonderful) intuition in [15], the neighborhood of the relative equilibrium in [8], the neighborhood of the Eight in [10] (relying on the numerical computation of the monodromy matrix of the Eight), the whole P_{12} family, as well as generalizations to other relative equilibria and other families of choreographies in [9] (the continuity of the families is only known numerically).

Problem sessions

Good references on the two-body problem are Arnold's book or, for more details, Albouy's lectures [1].

Various properties of the global evolution of the flow of the N-body problem are studied in [6], for example.

The given problem on periodic orbits in a perturbed Kepler problem is inspired by both Moser's paper [18] and [12] (which aims at finding quasiperiodic orbits). The book [4] gives a good introduction to perturbation theory.

The shape sphere is described in full details in the first 10 pages of [17].

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