Alberto Abbondandolo

Periodic orbits of exact magnetic flows on surfaces

I will discuss the question of existence and multiplicity of periodic orbits of exact magnetic flows, with special emphasis on surfaces and on energy levels below the Mañé critical value.

Alessandra Celletti

"From symplectic to conformally symplectic KAM theory with applications to Celestial Mechanics"

I will present some stability results based on KAM theory. I will consider both symplectic and conformally symplectic systems (either maps and flows). The proof of the KAM theorem is constructive and it gives very efficient algorithms to estimate the breakdown threshold of invariant tori. Applications to the (conservative and dissipative) standard map as well as to model problems in Celestial Mechanics are provided. Most of these works are done in collaboration with R. Calleja and R. de la Llave.

Angular momentum and Horn's problem

Alain Chenciner

The *central* configurations of n point masses in the euclidean space E are those configurations

$$x = (\vec{r}_1, \cdots, \vec{r}_n) \in E^n$$

which, if released without initial velocity, homothetically collapse on their center of mass when submitted to Newtonian attraction. For example, Lagrange has proved that the only non collinear central configuration of 3 positive masses is the equilateral triangle.

Such configurations are known (see [3, 4]) to admit periodic rigid motions, which necessarily take place in an euclidean space E of even dimension $2p$ and are of the form

$$\vec{r}_i(t) = e^{\omega t J}\vec{r}_i(0)), \quad i = 1, \cdots n,$$

where J is a complex structure on E compatible with the euclidean structure, that is an isometry such that $J^2 = -Id$. The angular momentum bivector of such a motion defines, via the euclidean structure, a J-skew-hermitian endomorphism $\mathcal{C}$ of E of the form $\omega(S_0J + JS_0)$, where the symmetric non negative $2p \times 2p$ matrix S_0 is the *inertia* matrix of the configuration $x(0)$ and ω is a real frequency. Replacing $\mathcal{C}$ by $\frac{1}{\omega}J^{-1}\mathcal{C}$, this leads to the following purely algebraic question :

Let S_0 be a symmetric non negative $2p \times 2p$ matrix; what is the image of the mapping $\mathcal{F}$ which, to each J, associates the ordered spectrum $\{\nu_1 \geq \nu_2 \geq \cdots \geq \nu_p\}$ of the J-hermitian matrix $S_0 + J^{-1}S_0J$, considered as a complex $p \times p$ matrix ?

On the other hand, Horn's problem (now solved independantly by Klyashko and by Knutson and Tao) asks for the possible spectra of matrices of the form $C = A + B$, where A and B are complex hermitian (or real symmetric) with given spectra.

Introducing two Horn's problems, one in dimension p and one in dimension $2p$, one proves that the image of $\mathcal{F}$ is a convex polytope which can be described. The precise result was conjectured in [1] and proved there in case $p = 2$; the general case was proved in [2].

Moreover, to this polytope are associated subpolytopes whose faces correspond to the only values of the angular momentum for which bifurcations could occur to families of quasi-periodic relative equilibria with *balanced* configurations (see [4]; these are the *configurations équilibrées* of [3]).

References

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[3] Alain Albouy & Alain Chenciner *Le problème des n corps et les distances mutuelles*, Inventiones Mathematicæ, **131**, pp. 151-184 (1998)

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James Montaldi

Classification of symmetries of planar choreographies

Abstract:
This classification involves two aspects: firstly a group-theoretic approach finding all possible symmetry groups of (simple) planar choreographies, and secondly a topological aspect classifying the connected components of the space of choreographies which each admissible symmetry group. I will endeavour to give a flavour of both. This is joint work with Katrina Steckles.

MASLOV INDEX AND SOME QUESTIONS OF DYNAMIC STABILITY

DANIEL OFFIN

Abstract. Several examples from stability of dynamical systems, have common topological features connected with the calculation and interpretation of the Maslov index.
Using an elementary notion of rotation of Lagrangian subspaces, we consider questions in parametric stability for time dependent Hamiltonian systems, and develop a method based on this notion, to determine the resonance regions in the parameter space. Next we consider the notion of minimum distance lines, in Jacobi metric for Hamiltonian systems of kinetic plus potential type. All such examples are generically hyperbolic on their energy surface. Several examples are shown to have this common feature. The family of homographic periodic solutions in the parallelogram four body problem which keep a rhombus configuration, are shown to be minimum distance liines after reduction by a rotational synmmetry. The instability of this family is thereby expressed in geometric terms.
Finally we consider an interesting family of mountain pass critical curves in Henon-Hieles Hamiltonian, with two degrees of freedom. We discuss the evolution of wave front sets along brake orbits in this example, and relate this to the determination of stability type for families of mountain pass lines.

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Rafael Ortega

A fixed point theorem for area-preserving maps

Abstract: A homeomorphism of the open disk has at least one fixed point if
it preserves orientation and area. This is an old result and we will review it. Later we will discuss a slight improvement and some consequences in stability theory.

Gabriella Pinzari

A full symplectic reduction for the planetary problem which preserves the symmetry by reflections, with an application

We shall present a set of canonical coordinates for the planetary system which is fitted to the its quasi-intergrability, reduces completely the number of degrees of freedom, keeps the symmetry due to reflections and is regular for zero inclinations. We shall show how this system of coordinates can be used to prove the existence of full-dimensional quasi-periodic motions, away from almost circular, co-planar trajectories. This is a new strategy for finding stable motions, independent of the program outlined by V.I.Arnold in 1963, and next developed by Arnold himself, J. Laskar, P. Robutel, M. Herman, J. F\'ejoz, L. Chierchia and the author, where full-dimensional tori were constructed closely to circular, co-planar trajectories. This new approach also allows to infer continuity between spatial and planar motions.

Silvia Sabatini

Hamiltonian actions of tori and semi-toric systems on 4-dim compact symplectic manifolds

On a compact symplectic manifold of dimension 4 there are two related classification theories: that of compact Hamiltonian S^1 spaces by Ahara-Hattori, Audin and Karshon, and that of compact symplectic toric manifolds by Delzant.
More recently, Pelayo and Vu Ngoc classified semi-toric systems on 4-dimensional (not necessarily compact) symplectic manifolds, namely Hamiltonian (S^1 x R) -actions with

special singularities.

In this talk I will explore some interactions among these three theories, and some

consequences of the compactness assumption in the semi-toric category.

(Based on the article``From compact semi-toric systems to Hamiltonian S^1-spaces" by S. Hohloch, D. Sepe and myself, and on ongoing joint work with S. Hohloch, D. Sepe and M. Symington).

Sergei Tabachnikov

Title: Frieze patterns and discretization of a Virasoro orbit

Abstract: Frieze patterns are beautiful combinatorial objects, introduced and studied by Coxeter and Conway in the 1970s. Recently they have attracted much attention due to their relation to the theory of cluster algebras. There is an intimate relation between three spaces: the space of closed frieze patterns, the moduli space of polygons in the projective line, and the space of 2nd order linear difference equations with skew-periodic solutions.

In this talk, I shall illustrate this relation by interpreting frieze patterns as a discrete version of a coadjoint orbit of the Virasoro algebra. The canonical (pre)symplectic form on the space of frieze patterns, associated with its cluster structure, is a discretization of the Kirillov symplectic form. I shall define a continuous version of frieze patterns as a certain PDE and relate it to conformal metrics of constant curvature in dimension 2. No preliminary knowledge of the above mentioned material is assumed.

Susanna Terracini

Parabolic trajectories in the N-body problem

We seek parabolic trajectories having prescribed asymptotic directions at infinity and which, in addition, are Morse minimizing geodesics for the Jacobi metric. Such trajectories correspond to saddle heteroclinics on the collision manifold, are structurally unstable and appear only for a codimension-one submanifold of such potentials. We give them a variational characterization in terms of the behavior of the parameter-free minimizers of an associated obstacle problem. We then give a full characterization of such a codimension-one manifold of poten-tials and we show how to parameterize it with respect to the degree of homogeneity. This is a joint work with V. Barutello and G. Verzini