Practical Applications of Monte Carlo Methods in Financial Institutions

Sommersemester 2017

Seminar Monte-Carlo-Methoden in der Finanzpraxis
Cologne, Germany, April 21, 2017

Contents

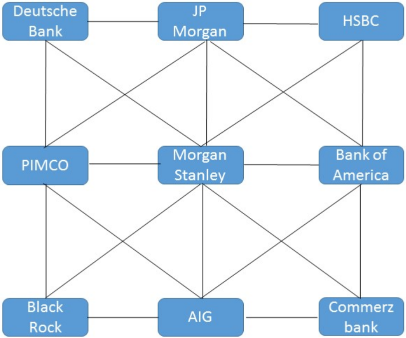
Over The Counter (OTC) Derivative Trading

B ö r s e

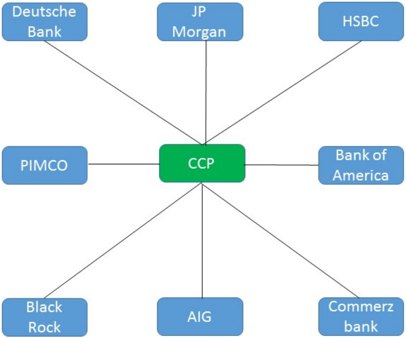
O T C

Find a fair derivative value (fair for both counterparties)!

Bilateral Trading vs. Central Counterparty (CCP) Clearing



Bilateral Trading



CCP Clearing

Infrastructure and Regulation

EMIR (Europe) and Dodd Frank Act (USA) aim at

- Enhancing transparency
- Reducing counterparty risk
- Reducing operational risk
- Increasing market stability
- Reduction in systemic risk

to avoid financial crisis in the future

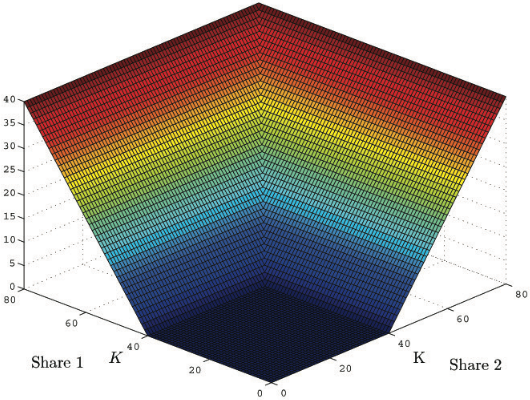
Financial Instruments

OTC derivatives:

1. Interest rate swaps
2. Credit default swaps
3. Plain vanilla options (with an early exercise feature)
4. Exotic options (any imaginable product)
5. among others

Rainbow Option I

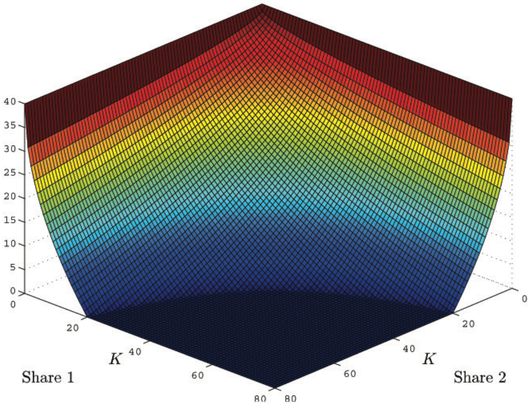
Max Call on 2 Shares



Payoff=max$\{0$,max(S_1, S_2)-Strike$\}$

Rainbow Option II

Geometric Average Put on 2 Assets



Payoff=max$\{0,\text{Strike-}\sqrt{S_1 S_2}\}$

Pricing Derivatives

$$\underbrace{\text{Methods for pricing financial derivatives}}_{\swarrow \searrow}$$

Numerical (Determinstic) Methods:

- Binomial Tree
- Finite Differences
- Finite Elements Methode
- Finite Volumes
- Analytical Solutions

Monte Carlo Methods:

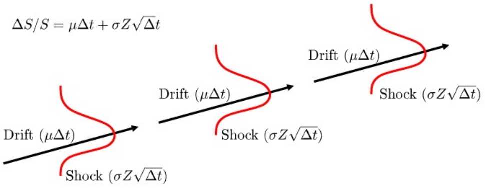
- Simulations
- Bounds
- Stochastic Grids
- Regression-based Methods

Integration of the underlying SDE I

Geometric Brownian Motion:	$S_t = S_{t-1} + \int_{t-1}^{t} rS_t ds + \int_{t-1}^{t} \sigma dW_s$
Short:	$dS_t = rS_t dt + \sigma S_t dW_t$
Analytical Solution:	$S_t = S_{t-1} exp((r - 0.5\sigma^2)t + \sigma W_t)$

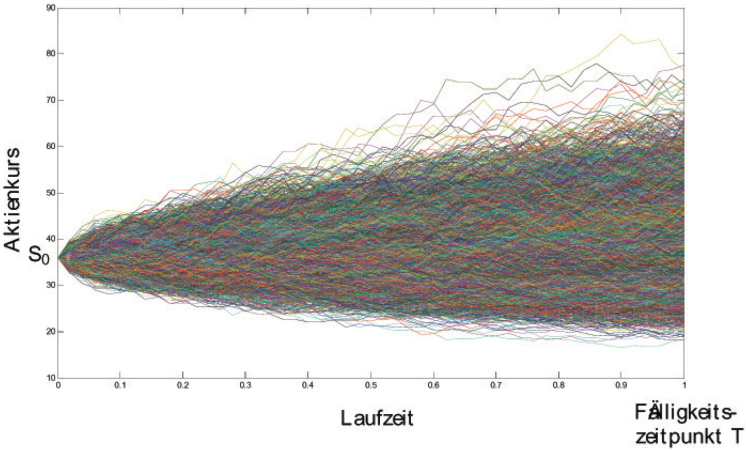
Path Generation: $W_t = W_{t-1} + dW_t, W_0 = 0, dW_t \sim N(0, dt)$

$Z \sim N(0, 1) \Rightarrow Z\sqrt{dt} \sim N(0, dt)$



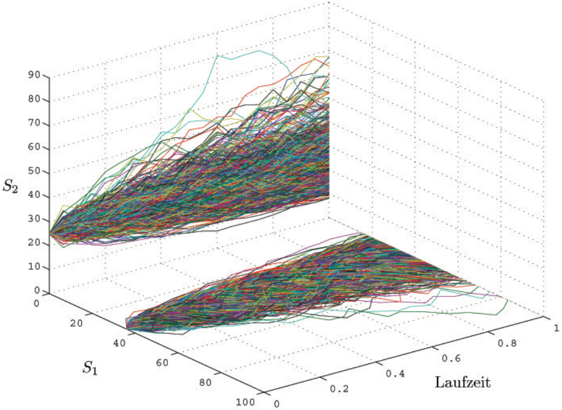
Integration of the underlying SDE II

Simulation of 10,000 asset paths

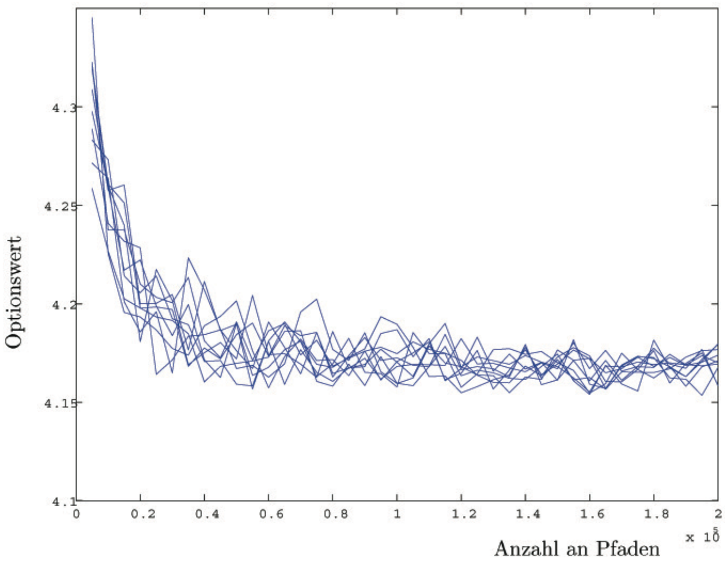


Integration of the underlying SDE III

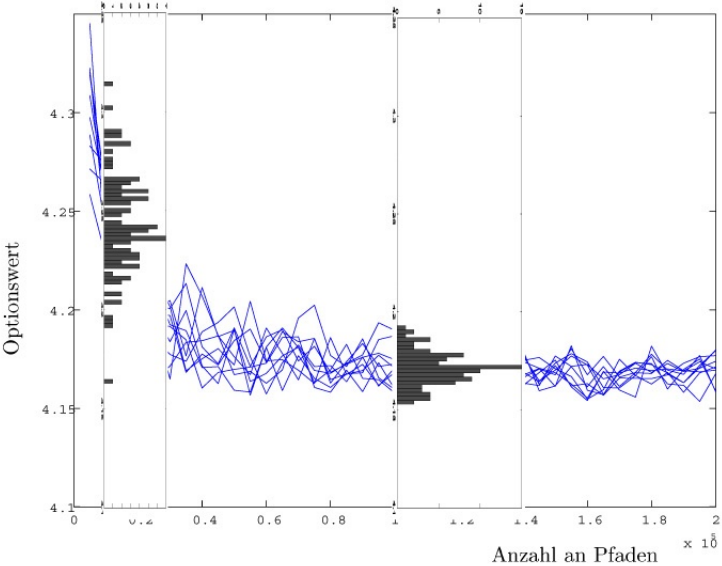
Simulation of 2,000 paths of two correlated stocks



Convergence I



Convergence II



Convergence III

Improve convergence by

- Develop efficient methods for pricing financial derivatives
- Random numbers with low discrepancy (Quasi Monte Carlo)
- Variance reduction techniques