Point Process Models

Quantitative Risk Management

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- POT model as a starting point for developing more dynamic descriptions
- POT model subsumes the models for maxima and the GPD models

Threshold exceedances for strict white noise

- Strict white noise process $(X_i)_{i \in \mathbb{N}}$ representing financial losses
  - iid
  - $\mathbb{E}[X] = 0$
  - $\sigma^2 = \mathbb{E}[X^2] < \infty$

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  - $\mathbb{E}[X] = 0$
  - $\sigma^2 = \mathbb{E}[X^2] < \infty$
- Common loss distribution $\in$ MDA(H_ξ), that means

$$\lim_{n\to\infty} F(c_n x + d_n) = H_\xi(x))$$

$$\Rightarrow \lim_{n\to\infty} nln(F(c_n x + d_n)) = ln(\lim_{n\to\infty}(F^n(c_n x + d_n)) = ln(H_\xi(x)).$$

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- Sequence of thresholds $(u_n(x))$ defined by $u_n(x) := c_n x + d_n$, so that

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and with $\lim_{y\to 1} -ln(y) = 1 - y$ it follows

$$\lim_{n\to\infty} n\bar{F}(u_n(x)) = \lim_{n\to\infty} -nln(F(u_n(x))) = -ln(H_\xi(x))$$

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- $N_{u_n(x)} := \#\{i \in \{1, ..., n\} : X_i > u_n\}$ is the number of exceedances of $u_n(x)$ by $X_1, ..., X_n$ with $N_{u_n(x)} \sim B(n, \bar{F}(u_n(x)))$. $N_{u_n(x)}$ converges to a Poisson RV with mean $\lambda(x) = -ln(H_\xi(x))$ as $n \to \infty$.

Poisson Point Process

Definition

Let $X_1, ..., X_n$ *be a sequence of RV's on some state space* $\mathcal{X}$. *The point process* $N(\cdot)$, *defined as* $N_n(A) = \sum_{i=1}^{n} \mathbb{1}_{\{X_n \in A\}}$ *for any set* $A \subset \mathcal{X}$, *is called a Poisson point process (or Poisson random measure) with intensity measure* Λ *if the following two conditions are satisfied*

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(a) *For $A \subset \mathcal{X}$ and $k \geq 0$,*

$$P(N(A) = k) = \begin{cases} e^{-\Lambda(A)} \frac{\Lambda(A)^k}{k!} & , \Lambda(A) < \infty \\ 0 & , \Lambda(A) = \infty \end{cases}$$

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(b) *For any $m \geq 1$, if $A_1, ..., A_m$ are mutually disjoint subsets of $\mathcal{X}$, then the RVs $N(A_1), ..., N(A_m)$ are independent.*

Intensity measure and intensity function

The Intensity or mean measure $\Lambda(\cdot)$ has the following properties:

- $\Lambda(A) = \mathbb{E}[N(A)]$ for $A \subset \mathcal{X}$,
- $\Lambda(A) = \int_A \lambda(x)\mathrm{d}x$, where $\lambda(x)$ is the intensity function or rate and $A \subset \mathcal{X}$.

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- $N_n(A) = \sum_{i=1}^{n} \mathbb{1}_{\{Y_{i,n} \in A\}}$ is the point process of exceedances of the threshold u_n with state space $\mathcal{X} = (0, 1]$.

Asymptotic behaviour of the point process of exceedances

Hence we can say
$N_n(\cdot)$ converges in distribution on $\mathcal{X}$ to a Poisson point process $N(\cdot)$

$$\lim_{n\to\infty} \mathbb{P}[N_n(A) \leq k] = \mathbb{P}[N(A) \leq k], k \in \mathbb{N}.$$

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The Poisson point process $N(\cdot)$ has intensity measure $\Lambda(\cdot)$ satisfying
$\Lambda(A) = (t_2 - t_1)\lambda(x)$ for $A = (t_1, t_2) \subset \mathcal{X}$, where $\lambda(x) = \lambda = -ln(H_\xi)$

$$\Rightarrow \mathbb{E}[N_n(A)] \to \mathbb{E}[N(A)] = \Lambda(A) = (t_2 - t_1)\lambda(x)$$

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We call this limiting process a *homogeneous Poisson process* with intensity λ.

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Let n be a fixed large sample size and $u = c_n y + d_n$ a fixed high threshold for some fixed y. Assume

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Replace c_n and d_n by $\sigma > 0$ and μ to get a Poisson process with rate $-ln(H_{\xi,\mu,\sigma}(u))$.
$\Rightarrow$ Exceedances of the level $x \geq u$ occur according to a Poisson process with rate $-ln(H_{\xi,\mu,\sigma}(x))$. That shows us the relation between the GEV model for block maxima and this model.

The Peaks over Threshold (POT) Model

- asymptotic model for threshold exceedances in iid data or data from a process with extremal index $\theta = 1$ with the following assumptions:

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  (a) exceedances occur according to a homogeneous Poisson process in time,
  (b) excess amounts above thresholds are iid and independent of exceedance times,
  (c) excess amounts are generalized Pareto distributed.
- Sometimes it is called *marked Poisson point process* where
  (i) Exceedance times constitute the points and
  (ii) GPD-distributed excesses are the marks.

Two-dimensional Poisson formulation

The POT model can also be described as a (non-homogeneous) *two-dimensional Poisson* point process with points (t,x) with t time and x magnitude of exceedances. Assume:

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- u is a high threshold,
- $\mathcal{X} = (0, 1] \times (u, \infty)$,
- the point process defined by $N(A) = \sum_{i=1}^{n} \mathbb{1}_{\{(i/n, X_i) \in A\}}$ is a Poisson process with intensity at a point(t, x) given by

$$\lambda(x) = \begin{cases} \frac{1}{\sigma}\left(1 + \xi \frac{x-\mu}{\sigma}\right)^{-1/\xi - 1} & , (1 - \xi(x - \mu)/\sigma) > 0 \\ 0 & , \textit{otherwise}. \end{cases} \quad (1)$$

Two-dimensional Poisson formulation

For a set of the form $A = (t_1, t_2) \times (x, \infty) \subset \mathcal{X}$, the intensity measure is

$$\Lambda(A) = \int_{t_1}^{t_2} \int_{x}^{\infty} \lambda(y) \mathrm{d}y \mathrm{d}t = -(t_2 - t_1) ln(H_{\xi,\mu,\sigma}(x)).$$

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$\Rightarrow$ For any $x \geq u$, the implied one-dimensional process of exceedances of the level x is a homogeneous Poisson process with rate
$\tau(x) := -ln(H_{\xi,\mu,\sigma}(x))$.
The tail of the excess df $\bar{F}_u(x)$ can be calculated as the ratio of the rates of exceeding the levels $u + x$ and u. We obtain

$$\bar{F}_u(x) = \frac{\tau(u+x)}{\tau(u)} = \left(1 + \frac{\xi x}{\sigma + \xi(u - \mu)}\right)^{-1/\xi} = \bar{G}_{\xi,\beta}(x)$$

for a positive scaling parameter $\beta = \sigma + \xi(u - \mu)$.

Relation to the GEP and GEV models

- $\bar{G}_{\xi,\beta}$ is precisely the tail of the GPD model for excesses over the threshold u with the parameter β.

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- $\bar{G}_{\xi,\beta}$ is precisely the tail of the GPD model for excesses over the threshold u with the parameter β.
- The relation to the GEV model is shown through the following: Assume we have an event $\{M_n \leq x\}$ for some value $x \geq u$. In point process language that is an event that there are no points in the set $A = (0,1] \times (x, \infty)$.

$$\Rightarrow \mathbb{P}[M_n \leq x] = \mathbb{P}[N(A) = 0] = e^{-\Lambda(A)} = H_{\xi,\mu,\sigma}(x) \geq u.$$

This is precisely the GEV model for maxima of n-blocks.

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$\Rightarrow$ The intensity (1) is often used as a framework to introduce covariate effects into extreme value modeling

Maximum likelihood method

For exceedance data $\{\tilde{X}_j : j = 1, ..., N_u\}$ the likelihood is

$$L(\xi, \sigma, \mu : \tilde{X}_1, ..., \tilde{X}_{N_u}) = e^{-\tau(u)} \prod_{j=1}^{N_u} \lambda(\tilde{X}_j).$$

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The intensity in (1) can therefore be written as

$$\lambda(x) = \lambda(t, x) = \frac{\tau}{\beta} \left(1 + \xi \frac{x-u}{\beta}\right)^{-1/\xi - 1},$$

with $\xi \in \mathbb{R}$ and $\tau, \beta > 0$, where

$$\tau = \tau(u) = -ln(H_{\xi,\mu,\sigma}(u)) \text{ and } \beta = \sigma + \xi(u - \mu)$$

Maximum likelihood method

The log-likelihood for the parameters ξ, σ, μ can be written as

$$ln(L(\xi, \sigma, \mu; \tilde{X}_1, ..., \tilde{X}_{N_u})) = lnL_1(\xi, \beta; \tilde{X}_1 - u, ..., \tilde{X}_{N_u} - u) + lnL_2(\tau; N_u).$$

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This is easier to calculate and shows the relation to the GPD and GEV model because

- L_1: likelihood for fitting a GPD to excess losses,
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First calculate ξ and β in a GPD-analysis and then τ by its MLE N_u to get the estimates of μ and σ.

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- Identify clusters by using the runs method for example.
- Identify the maximum excess in each cluster.
- Apply the POT model to these maxima only.

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but by neglecting the modeling of cluster formation we however cannot make more dynamic statements about the intensity of occurrance of threshold exceedances at any point in time.

POT analysis of AT&T weekly losses

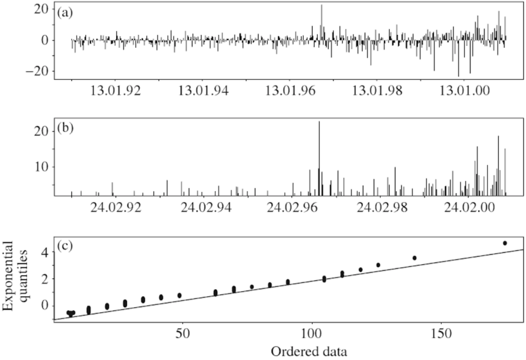

Figure: (a)Time series of AT&T weekly percentage losses from 1991 to 2000. (b)Corresponding realization of the marked point process of exceedances of the threshold 2.75%. (c)Q-Q plot of inter-exceedance times against an exponential reference distribution.

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By using the likelihood-method we get the parameter estimates

- $\hat{\xi} = 0.22$,
- $\hat{\mu} = 19.9$,
- $\hat{\sigma} = 5.95$,
- and the estimated exceedance rate

$$\hat{\tau}(2.75) = -ln(H_{\hat{\xi},\hat{\mu},\hat{\sigma}}(2.75)) = 102.$$

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We can also calculate for example $\hat{\tau}(15) = 2.50$ and get that losses exceeding 15% occur as a Poisson process with rate 2.5 per ten-year period.

Thank you for your attention!

Literature

Alexander J. McNeil, Ruediger Frey, Paul Embrechts (2015): *Quantitative Risk Management: Concepts, Techniques and Tools*: University Press Group Ltd