Zoran Nikolić, Christian Jonen, Chengjia Zhu

Robust Regression Technique in LSMC Proxy Modeling

Summary

In this paper we introduce the so-called robust regression technique into LSMC proxy modeling for life insurance companies. After describing the fundamental properties of the two methods that are applied here together for the first time, we compare the results of ordinary least squares (OLS) regression with an array of robust regressions applied to the portfolios of two German life insurers and one German health insurer. While choosing a suitable polynomial model is left to OLS regression for practical purposes, for a given polynomial model the robust regression method systematically yields better results than OLS. We extend the existing theoretical framework for robust regression by introducing asymmetric boundaries for the first time.

Introduction

There is a long history of beneficial transfer of ideas from financial mathematics into the field of life insurance valuation. The entire market-consistent valuation paradigm, which underlies the Solvency II framework, stems from the Black-Scholes approach to valuing uncertain (conditional) future cash flows.

Following the success of the Longstaff-Schwartz proposal for Monte Carlo valuation (F. Longstaff and E. Schwartz, 2001) of the continuation value of American-styled options, abbreviated Least Squares Monte Carlo (LSMC), various developments from regression analysis and statistics were introduced to the LSMC financial mathematics framework, among them, the robust regression technique. We propose transferring this approach to the field of life insurance valuation. Market-consistent valuation of a life insurance company consists in calculating the expected value of the future cashflows (premiums, benefits, etc.), which is quite similar to pricing a financial derivative. Since the insurance company[1] and its cashflows are vastly more complex than common financial derivatives, it is not immediately clear which methods from financial mathematics may have more and which less utility when applied to insurance companies. For this reason, each new method has to be separately analyzed.

Lately, an adaptation of the LSMC method originally developed for the pricing of American options has proved to be a remarkably stable approach to approximating the balance sheet items for life insurance companies. An introduction to LSMC as an aggregation technique used for risk capital purposes can be found in (DAV-Arbeitsgruppe *Aggregationstechniken*, 2015). The analysis in this article is a result of practitioners' research on the LSMC technique in the insurance context driven by issues encountered in the praxis. In order to have a good fit for all relevant risk factor combinations, usually very large fitting spaces, a high occurrence of unrealistic scenarios are allowed, which, may yield extreme cashflows for single fitting simulations. Although the core idea of the Least Squares Monte Carlo approach allows for single fitting points to be very inaccurate, for the actuarial projection tools that replicate the complexity of the profit sharing mechanism in an insurance company the "outlierness" of a simulation may become extreme. For that reason, it is natural to think about reasonable ways of assigning different weights to the regression residuals depending on their presumed reliability or on their impact on the 99.5th percentile.

Following up on this, we apply the robust regression technique, which is already a well-known and established technique in statistical modeling. We demonstrate that the transfer of the robust regression, as proposed in (Jonen 2011) for financial market instruments, systematically delivers improvements in the LSMC proxies for the insurance companies examined in the subsequent case study. We extend the existing academic framework of robust regression by introducing asymmetric boundaries in cases when residuals appear to be substantially heteroscedastic. In our study we perform only ex post analyses assuming that the polynomial model defined by OLS is the "true one". A process which would use the robust regression in the derivation of the terms of the polynomial proxy function still has to be defined. We believe further research with this and other techniques will eventually allow for a description of a mathematically sound, widely applicable and reliable framework for the derivation of Solvency II capital calculation proxies.

Brief Description of the LSMC Procedure as Currently Applied to Life Insurance Companies

In this section we outline the current state of the applications of the LSMC method in the insurance industry broadly following the model described in (Koursaris, 2011). For details regarding the application of LSMC for Solvency II purposes we refer to (DAV-Arbeitsgruppe *Aggregationstechniken*, 2015). It is important to understand that the original Longstaff-Schwartz idea referred to the continuation value of American options whereas, for our purpose —

[1] The insurance company is for this purpose understood as an "exotic derivative"

after the method has been transferred to the insurance industry — it becomes an approximation method for the expected present value of future cash flows between the company and its policyholders (Technical Provisions) or its shareholders (Own Funds) conditional on the state of risk factors included in the company's risk model. There are various efforts ongoing regarding a theoretical foundation for the application of the LSMC approach in the insurance industry. In (Beek, 2016) the continuity of the function

$$PV(RF_1, \dots, RF_D)$$

obtained via actuarial projection tools was shown, where $RF_1, \dots, RF_D$ are the initial states of risk factors to which the company is exposed and PV is the present value of the future cash flows between the company and its policyholders. The same holds for the present value of future cash flows to and from shareholders.

The practical LSMC implementations in companies largely follow (Koursaris, 2011). We describe the base setting employed in this study which, in principle, reflects this approach.

1. The fitting space for an LSMC regression is determined as a cube $I_1 \times \dots \times I_D$ defined by intervals $I_k \ni RF_k$ corresponding to single risk factors $RF_1, \dots, RF_D$, see Figure 1 for a two-dimensional representation. They are the regressors in the regression problem we are about to solve. Each interval has to be sufficiently wide so as to cover an area large enough to make reliable univariate predictions about this single risk factor. For instance, the fitting interval I_k may cover 99.9% of possible (i.e. real-world) outcomes of the risk factor RF_k. On the other hand, although most points around the vertices of the cube are extremely unlikely in the sampling from the joint distribution of all risk factors, a uniform fit across the entire space is chosen. One important reason for this lies in the fact that we do not know where the Value-at-Risk (VaR, 99.5th percentile) or any other risk measure will eventually lie. We rely on the theory of low-discrepancy sequences (Glasserman, 2003) to cover the fitting space in an optimum uniform manner.

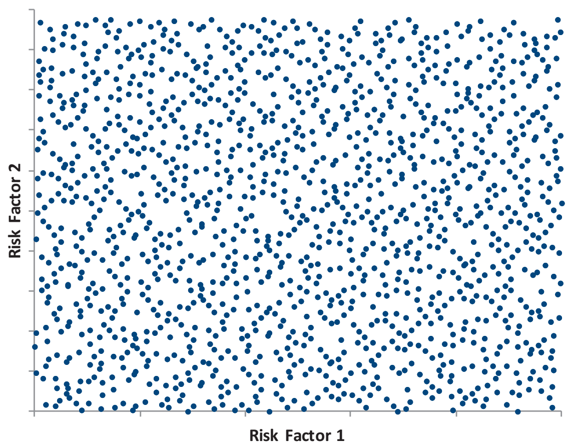
2. The fact that there is a uniform population of the fitting space leads necessarily to evaluations with the actuarial projection tool in a number of points which are highly unlikely, so by construction the fitting values are prone to extreme values. It is important to bear in mind that – contrary to financial options, which have plausible values for any development of the underlying – actuarial projection tools may face significant challenges with very extreme economic conditions. For instance, due to the projection periods of at least 50 years, in case of hyper-inflation scenarios the resulting future cash flows may become numerically out of reach. Furthermore, we have to account for the effects of restrictions imposed on management actions within actuarial projection tools, which imply practically unbounded losses for the company in any given year once the underlying economy is in a severe financial distress. This leads to a continuation of business activity after ruinous losses as if the company were still able to meet its obligations[2].

3. As in the original LSMC approach for each combination of risk factors — called outer scenario — two antithetic inner simulations are calculated instead of thousands of simulations which are necessary if we want to calculate the exact value. The inaccurate mean value of just two inner scenarios leads to the fitting point *PV*, representing a rough yet unbiased estimate for a certain quantity of the economic balance sheet such as the Technical Provisions or Own Funds Usually there are tens of thousands of outer scenarios, with each of them leading to one fitting point.

4. Using an approach from information theory a model is constructed which is used to represent the so-called proxy function, aiming at representing a functional relationship between expected present value of the quantity under consideration (see 3, and the risk factor values). This model becomes built up during the regression over the fitting points PV. A practical model function can be given by a simple linear combination of basis functions $\{\varphi_m(\cdot)\}_{m=1}^M$, i. e.

$$f(RF_1, \dots, RF_D) = \sum_{m=1}^{M} a_m \varphi_m(RF_1, \dots, RF_D) \quad (1)$$

Figure 1:
Fitting space for two risk factors as a cube $I_1 \times I_2$



with real numbers a_m. As far as basis functions are concerned, polynomials are a possible choice, but also trigonometric or radial functions. For the decision whether a model better describes the fitting points PV, ordinary least square (OLS) regressions are performed. An information criterion is used to decide whether the addition of a basis function is justified.

5. The ultimate test of the proxy quality consists in the out-of-sample validation. For this purpose, a number of exactly calculated values in outer scenarios not belonging to the original sample are compared to the proxy values $f(RF_1, \dots, RF_D)$. The set of these outer scenarios has to be sufficiently large to enable statistical analysis of the deviations.

6. As the final step, the polynomial derived by this method is used in the real-world evaluation of the risk-factors in order to reliably the VaR or any other risk measure.

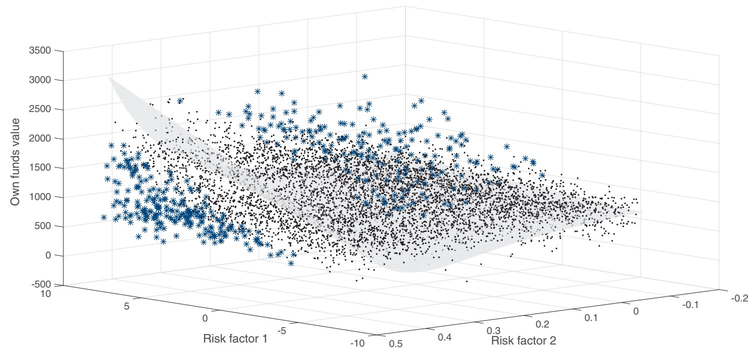
It is important to note that the number of scenarios and the number of basis functions, i.e., the complexity of the model function, have to be increased simultaneously in order to achieve convergence and a sufficient goodness-of-fit. Therefore, for a given accuracy, potentially an extended model has to be created in order to achieve the goals.

The subject of this analysis is the calculations of Own Funds proxy functions. The same approach can be applied to a much wider array of applications, such as dependency of the risk and capital figures from the assets or liabilities as well as management decisions (by fitting them as additional parameters simultaneously to the original risk model).

Brief Description of the Robust Regression Technique

Let us now describe the robust regression approach applied to insurance companies. First we take a closer look at the Own Funds proxy function calculated by OLS in Figure 2. On closer examination, it becomes clear that there are some points — namely the blue stars — that are far away from this proxy function. We call these points outliers which might destroy the structure of the assumed model function (1). This observation directly leads to robust regression, as robust regression is able to take outliers[3] into account and preserve the approximation quality even in the presence of data perturbations.

Figure 2:
Own Funds Proxy Function calculated by OLS



Having this motivation in mind, we suggest replacing the OLS problem by the following optimization problem to determine the coefficients of the model function (1):

$$\min_{a \in R^M} \frac{1}{N} \sum_{n=1}^{N} l(PV_n - f(RF_{1n}, \dots, RF_{Dn})) \quad (2)$$

with a suitably measurable loss function

$$l: R \to [0, \infty)$$

specified a priori; for $n = 1, \dots, N$, $RF_{1n}, \dots, RF_{Dn}$ are the N simulated risk factor vectors (outer scenarios) and PV_n are N sample points.

The minimization is performed over coefficients $a \in R^M$ of basis functions $\{\varphi_m(\cdot)\}_{m=1}^{M}$. In the following we denote the residuals by

$$r_n := PV_n - f(RF_{1n}, \dots, RF_{Dn}) \quad (3)$$

Let us make some assumptions on this loss function to guarantee the well-posedness of (2):

(L1) $l(\cdot)$ is a piecewise twice continuously differentiable function with $l(r) = 0$ if $r = 0$.
(L2) $l'(r)/r \geq 0$ and $l''(r) \geq 0$ for any $r \in R$.

In so doing, we consider (piecewise) convex loss functions such that the optimization problem (2) has a unique (global) minimizer. In this article we concentrate on the loss functions listed in Table 1, but we note that other robust loss functions might be selected as well.

Figure 3 shows the graphs of the OLS, Huber, Talwar and Jonen loss function for symmetric and asymmetric thresholds. As we can see, the key idea of robust regression is to give outliers less weight than the other points.

The first order condition of the minimization problem (2) becomes the following set of M equations:

[2] We remind that an implementation of benefit reductions or similiar actions- when a company is obviously insolvent, would not comply with regulatory modeling standards.
[3] Or any values that distort the regression, for instance in the VaR regions

$$\sum_{n=1}^{N} l'(r_n) A_{nm} = 0, m = 1, \ldots, M, \quad (4)$$

where $A_{nm} = \varphi_m(RF_{1n}, \ldots, RF_{Dn})$, $n = 1, \ldots, N, m = 1, \ldots, M,$ is the nm entry of the regressor (design) matrix $A \in R^{NxM}$.

This nonlinear system has to be solved by iterative solvers and we refer to (Jonen, 2011) for an efficient Newton-Raphson-based solver to tackle problem (4).

A vital next step is to discuss outlier detection procedures to determine the thresholds. Robust regression is often applied in a statistical context and reasonable distributions of the residual r_n in (3) are assumed. As we are not familiar with any distribution in our actuarial framework, we should work with empirical distributions as pointed out in (Jonen, 2011). By doing so, possible outlier detection procedures on the basis of empirical quantiles are given as follows:

1. Symmetric thresholds:

$$\begin{gathered} r_n^{help} = |r_n|, n = 1, \ldots, N, r = (r_1, \ldots, r_N) \\ r^{help} = sort(r^{help}) \\ \gamma_1 = r_{\lfloor \alpha N \rfloor}^{help}, 0 \ll \alpha \leq 1 \\ \gamma_2 = r_{\lfloor \beta N \rfloor}^{help}, \alpha \leq \beta \leq 1 \end{gathered} \quad (5)$$

2. Asymmetric thresholds:

$$\begin{gathered} r_n^{help} = r_n, n = 1, \ldots, N, r = (r_1, \ldots, r_N) \\ r^{help} = sort(r^{help}) \\ \delta_1 = r_{\lfloor \rho N \rfloor}^{help}, 0 \leq \rho \leq \tau \\ \delta_2 = r_{\lfloor \tau N \rfloor}^{help}, 0 \leq \tau \ll 1 \\ \delta_3 = r_{\lfloor \alpha N \rfloor}^{help}, 0 \ll \alpha \leq 1 \\ \delta_4 = r_{\lfloor \beta N \rfloor}^{help}, \alpha \leq \beta \leq 1 \end{gathered} \quad (6)$$

where $sort(\cdot)$ denotes the routine for sorting a vector in ascending order[4].

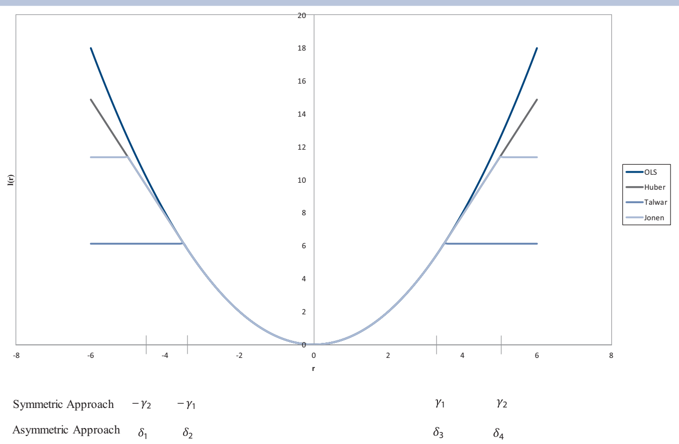
Following the symmetric approach we presume $(1-\alpha)100$ $((1-\beta)100)$ per cent of the data points to be (extreme) outliers a priori. On the contrary, following the asymmetric approach we assume that $\rho 100$ $(\tau 100)$ and $(1-\alpha)100$ $\left((1-\beta)100\right)$ per cent of the data points below and above the proxy function are (extreme) outliers, respectively. Note that the choices $\alpha = \beta = 1$ and $\rho = \tau = 0$ coincide with OLS.

By solving the OLS problem in a first step we are able to determine the empirical distribution of the residuals (3), and thus the transition points, by procedure (5) or (6); the OLS solution also serves as starting point for the iterative solution of (4). Figure 4 visualizes the outlier detection procedures (5) and (6) based on the calculated residuals (3) via OLS. The left histogram results from the OLS solution and is used for determining thresholds in (6); by taking absolute values of the underlying residuals, the thresholds in (5) are defined by considering the right histogram.

Table 1:
Several loss functions with symmetric and asymmetric thresholds for robust regression.

Name	$l(r)$ with symmetric thresholds	$l(r)$ with asymmetric thresholds
OLS	$0.5r^2$	$0.5r^2$
Huber	$\begin{cases} 0.5r^2 & \lvert r\rvert \leq \gamma_1 \\ \gamma_1\lvert r\rvert - 0.5\gamma_1^2 & \lvert r\rvert > \gamma_1 \end{cases}$	$\begin{cases} \delta_1 r - 0.5\delta_1^2 & r < \delta_1 \\ 0.5r^2 & \delta_1 \leq r \leq \delta_2 \\ \delta_2 r - 0.5\delta_2^2 & r > \delta_2 \end{cases}$
Talwar	$\begin{cases} 0.5r^2 & \lvert r\rvert \leq \gamma_1 \\ 0.5\gamma_1^2 & \lvert r\rvert > \gamma_1 \end{cases}$	$\begin{cases} 0.5\delta_1^2 & r < \delta_1 \\ 0.5r^2 & \delta_1 \leq r \leq \delta_2 \\ 0.5\delta_2^2 & r > \delta_2 \end{cases}$
Jonen	$\begin{cases} 0.5r^2 & \lvert r\rvert \leq \gamma_1 \\ \gamma_1\lvert r\rvert - 0.5\gamma_1^2 & \gamma_1 < \lvert r\rvert \leq \gamma_2 \\ \gamma_1\gamma_2 - 0.5\gamma_1^2 & \lvert r\rvert > \gamma_2 \end{cases}$	$\begin{cases} \delta_2\delta_1 - 0.5\delta_2^2 & r < \delta_1 \\ \delta_2 r - 0.5\delta_2^2 & \delta_1 \leq r < \delta_2 \\ 0.5r^2 & \delta_2 \leq r \leq \delta_3 \\ \delta_3 r - 0.5\delta_3^2 & \delta_3 < r \leq \delta_4 \\ \delta_3\delta_4 - 0.5\delta_3^2 & r > \delta_4 \end{cases}$

Figure 3:
Loss functions for robust regression based on a symmetric and asymmetric approach.



Case Study for Life and Health Insurance Companies

The focus of our numerical investigations is put on two German life insurers' portfolios A and B as well as a German health insurer's portfolio C.

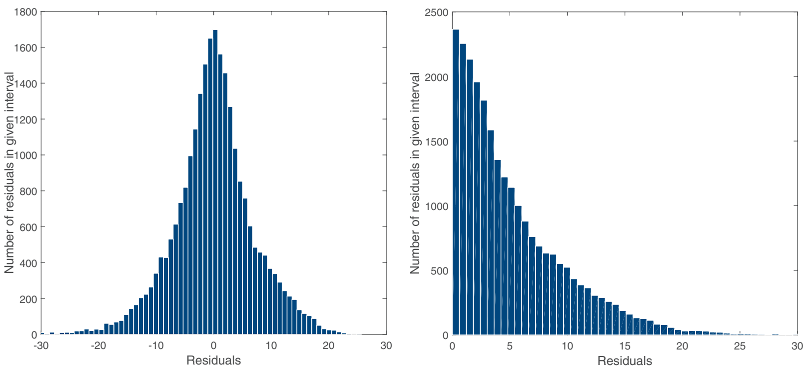
Scope and Framework

As stated above, we begin with the OLS regression to determine the model with polynomial basis functions for each of the insurance companies A, B and C. Only after deciding on the model do we perform additional robust regressions.

Current field experiences show significant impact of external parameters, such as market conditions, modeling of risk factors, robustness of the underlying actuarial projection tool or the choices

[4] "<<" denotes "truly less than"

Figure 4:
Empirical distribution of (absolute) residuals on the left (right) resulting from an OLS solution.



in the economic scenarios generator (ESG) parameterization on the goodness of fit . While our basic setting refers to symmetric thresholds, for companies B and C an additional robust regression analysis with asymmetric thresholds is performed.

Overall, two different settings are considered for this report:

- **Basic Setting** for all three companies A, B and C
- **Alternative Setting** for company B, where the functionalities in the actuarial projection tool, the market conditions and the risk-neutral ESG calibration are amended for a different valuation period

Quality Criteria for the Comparison

One of the core ideas of the LSMC method for insurance companies consists in a good fit across the entire fitting space. But this means some compromises have to be made and the regression does not put a particular weight on the VaR neighborhood, i.e. the regions of risk factor realizations leading to the 99.5th percentile of the Own Funds losses, which is the relevant measure for Solvency II. In order to compare the proxies for different robust regression proxies, a proper quality criterion needs to be defined.

In this study a large number of explicitly created out-of-sample scenario sets are used as reference values for the goodness-of-fit. These scenarios stem from the VaR neighborhood as defined by the OLS regression. They contain sufficiently many inner simulations for a good Monte Carlo estimate. Based on the proxy functions derived by running both methods, OLS and robust regression, we calculate the estimated SCR – 99.5th percentile of losses – as the distance between the base point (proxy intercept) and VaR. If a proxy delivers a closer approximation to the true SCR, which is based on the VaR calculated with explicit out-of-sample scenarios, it is considered to be more accurate and the underlying regression is hence preferred.

Results

In the following case studies the presented figures are (scaled) SCR deviations[5] : Proxy estimates of the SCR are subtracted from the SCR calculated explicitly with out-of-sample scenarios in the VaR neighborhood.

We start our numerical investigations by comparing the estimated SCR resulting from OLS and different robust regressions for the insurer A.

The polynomial derived by the OLS approach overestimates the SCR for company A. Table 2 shows the range of the possible improvements for insurer A with different α and β as parameters[6]. For instance, the approximation is already improved from 30.3 to 23.8 by introducing the outlier parameter α =0.95 and leaving the extreme outlier parameter β=1 unchanged. That means 5% of the data points are taken into account as outliers.
It becomes clear that, with the chosen α and β parameters, the estimated SCR with robust regression is always more accurate than the one with the OLS method.

Next we evaluate the same metrics and compare figures for insurer B. Table 3 shows the results calculated by the symmetric approach (5) for several combinations of α and β.

Contrary to the same analysis for insurer A, the accuracy deteriorates when robust regression is used. Although the observation might seem unexpected at first glance, the comparable inaccuracy can be explained as we will now see.

To start, let us look at the empirical distribution of the residuals (3) plotted in Figure 5.

While the distribution for insurer A is quite symmetric, we observe a left-skewed distribution for insurer B.

This motivates the use of our proposed asymmetric approach (6) in order to give the fitting points that are far away below and above the proxy function less weight. It is important to bear in mind that, considering the absolute residuals within the symmetric approach (5), the detected outliers for a left-skewed distribution would be mostly the points with negative residual values, i.e., the points below the proxy function; and for a right-skewed distribution vice versa. Thus, using the symmetric approach for an underlying skewed residual distribution leads to an indirect one-sided reduction of the weight for fitting points.

[5] The figures in the surveys are scaled with a factor for each company. An error of zero thus implies a perfect fit
[6] Figures result from the Talwar function in case of $\alpha \geq \beta$.

Figure 5:
Empirical distribution of residuals (3) calculated by OLS for insurer B.

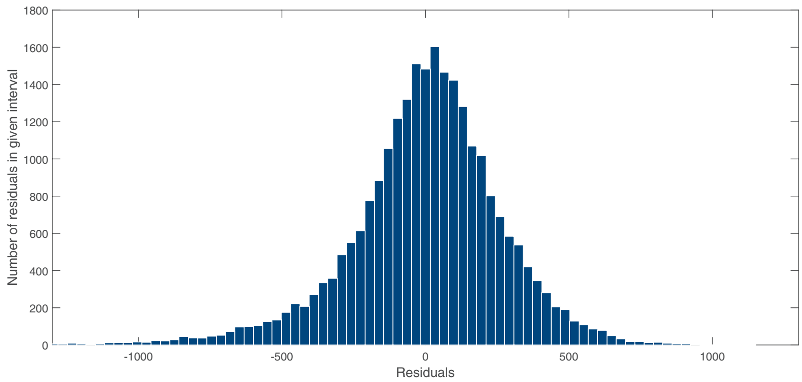
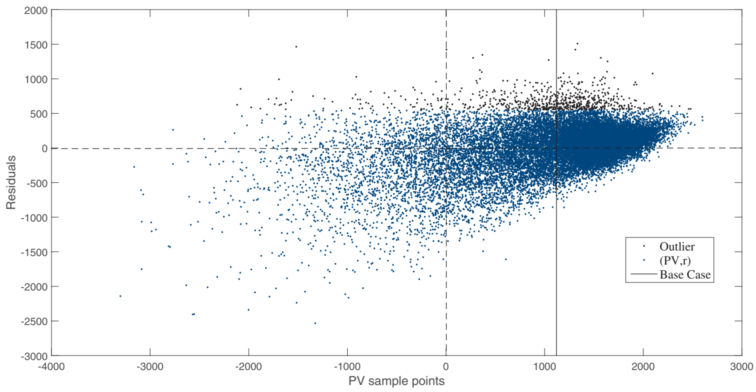


Figure 6:
PV sample points versus residuals for insurer B.



By analyzing figures in Table 4 we come to the conclusion that it is beneficial to give fitting points with high positive residual values less weight than the other points. In other words, an application of α and β leads to more accuracy in the SCR estimation.

After this vital observation of the empirical distribution, let us now extend our research by plotting the PV sample points PV_n versus the residuals r_n, see Figure 6.

We see increasing dispersion with decreasing PV values and, especially in the lower left-hand corner beginning with the base case line, a number of negative PV points with large negative residual values[7] . For an insurer, these points represent extreme stresses and therefore high losses.

We activate positive but not negative thresholds and by doing this we are able to introduce a stronger focus on these scenarios. This is reasonable, since we are primarily interested in goodness-of-fit in stressed regions as they define the VaR. As the fitting points in the positive region and close to the base case lie tightly around the zero line of the residuals, the proxy function shows robustness against this approach in these regions. Consequently, it reacts more sensitively in the negative region due to the high variance. The black points in Figure 6 indicate the outliers in our proposed asymmetric approach. With this methodology we can significantly increase the accuracy of the SCR estimation by giving points with high positive residuals less weight than the other points. As shown in Table 5, robust regression delivers comparable improvements as in the previous case study for company A when we allow asymmetric thresholds.

We achieve similar results by allowing a moderate threshold on the left-hand side, but see slightly slower convergence with decreasing combinations of α and β in Table 6.

To sum up, the key for successful implementation in this case study for insurer B is to reject the original idea of robust regression and to formulate an optimization problem allowing for adequate consideration of extreme stresses. By doing so, we are able to obtain a more accurate proxy function in regions which are most important for VaR estimates.

As the final step for insurer B, we perform the same exercise for insurer B with the alternative setting and a different choice of parameters. By using asymmetric thresholds we observe a drift of the proxy function away from the origin despite the described robustness in the positive region above, i.e., the fit of the base case without any stress deteriorates. In order to avoid this side effect, we should work with an intercept constraint to restrain the proxy function for the base case, as done in this case study.

The outcomes as shown in Table 7 and Table 8 confirm that the Jonen approach remains robust even in a changed environment[8]. Analogous to the previous calculations performed with the basic setting, the decision of proper values for α and β is a crucial part of the process. Once they are determined properly, the

[7] Base case means Own Funds without any stresses.

Table 2: Estimated SCR error for insurer A based on symmetric approach (5) with several thresholds.

		β										
		1	0.995	0.99	0.985	0.98	0.975	0.97	0.965	0.96	0.955	0.95
α	1	30.3	20.3	14.5	16.1	15.2	12.4	11.7	11.3	10.0	6.7	6.2
	0.95	23.8	17.8	14.2	15.1	14.5	12.0	10.9	11.1	9.9	6.6	6.2
	0.9	21.7	16.3	13.0	13.3	12.4	11.6	9.6	9.8	8.4	6.5	5.1
	0.85	20.2	15.2	12.3	12.1	11.4	10.8	8.7	8.4	6.5	6.9	3.4
	0.8	19.4	14.3	11.6	11.2	10.6	9.5	7.7	7.8	4.7	5.1	1.8
	0.75	18.8	13.8	11.4	10.8	9.7	9.0	7.8	6.8	4.1	4.4	0.9
	0.7	18.5	13.6	11.4	10.6	9.5	8.7	7.3	6.4	4.4	3.3	0.3
	0.65	18.2	13.2	11.1	10.5	9.4	8.6	6.9	6.4	4.6	3.2	0.7
	0.6	17.9	13.1	11.1	10.6	9.3	8.5	7.0	6.4	4.6	3.1	0.7
	0.55	17.7	12.9	10.9	10.3	9.1	8.2	6.7	6.0	4.5	3.0	0.1
	0.5	17.7	12.9	10.9	10.4	9.2	8.3	7.0	6.0	4.0	2.2	-0.1

Table 3: Estimated SCR error for insurer B based on symmetric approach (5).

		β				
		1	0.99	0.98	0.97	0.96
α	1	-72.1	-81.5	-89.6	-93.8	-99.6
	0.92	-80.2	-87.2	-92.6	-98.0	-102.1
	0.84	-85.6	-92.0	-98.3	-102.2	-107.3
	0.76	-86.5	-92.7	-98.6	-102.8	-108.3
	0.68	-85.4	-91.7	-97.4	-101.4	-107.5

Table 4: Estimated SCR error for insurer B based on an asymmetric approach.

β = 1 ρ = 0		τ		
		0	0.05	0.1
α	1	-72.1	-83.5	-98.2
	0.95	-62.5	-75.3	-91.8
	0.9	-51.5	-64.4	-83.3

proxy function can be improved significantly.

Finally, we perform similar assessments with the Jonen approach for health insurer C. Although the nature of the business in terms of management actions and actuarial formulae driving the cash flows of a health insurer are different from those of a life insurer, we can observe comparable outcomes produced by robust regression based on a symmetric approach (5), see Table 9.

Even though the empirical distribution of residuals is quite symmetric, we are able to get further improvements by applying the asymmetric approach (6) as shown in Table 10. Due to the underlying symmetry, we increase the number of outliers on both sides for the thresholds α and τ simultaneously. We see a significant reduction of the estimated SCR error for combinations below the diagonal in Table 10, meaning that the positive thresholds are more relevant than the thresholds for negative residual values. Referring to the previous case study for insurer B – but having the symmetry in mind – we reinforce the influence of the stress region within the optimization problem (2) by activating thresholds in that way. For extreme outliers, our numerical investigations show robustness against slight asymmetric values such that other combinations of β and ρ lead to similar improvements.

We believe that the robust regression technique or a similar approach can be applied to a wide variety of insurance companies covering different assumptions within actuarial projection tools.

Conclusion

We have demonstrated that the robust regression in the extended Jonen approach with asymmetric thresholds can significantly improve the quality of LSMC proxy models used for Solvency II capital assessments.

We have furthermore introduced a novel approach with asymmetric thresholds in the robust regression framework which helped us improve the fits of all companies in this case study.

In case of symmetric distribution, two parameters are sufficient to achieve a result superior to OLS. For skewed distributions, the asymmetric outlier detection procedure (6), with two thresholds for positive and two for negative residuals, is the more efficient one.

[8] Functionalities in the actuarial projection tool, the market conditions and the risk-neutral ESG calibration as defined above.

Obviously, we have presented just one possible test setting using only two regression approaches — albeit sophisticated ones — and it is logical to ask whether any other regression approach may yield comparable results.

We know that the problem of replicating the expected present value of complex cash flows from actuarial projection tools is a different problem to the one posed in the context of American option pricing. For our problem we expect a further development of insurance-specific efficient regression methods in the future.

Moving on, the information theory application in the standard LSMC setting relies on OLS regression. In our study the model selection procedure is based on OLS regression and its underlying textbook assumptions. It would certainly be worthwhile investigating how to choose a reliable model in a heteroscedastic environment and, furthermore, whether it is possible to choose a model tailored to robust regression instead of the OLS in a computationally feasible manner.

We are aware that, for measuring the goodness-of-fit, we employ the exact calculation based on the results of the OLS regression and not the robust regression. One has also to ask whether the VaR fluctuation for different robust regression proxies can have a material impact, i.e., whether, in an iterative process of conducting exact Monte Carlo valuation for the VaR neighborhood of robust regression proxies, the resulting SCR would change significantly or not.

There are still plenty of other statistical methods which should be tested in our valuation framework. Since the question of SCR calculation for insurance companies is a particular one with a high degree of complexity, potentially some new, innovative approaches will even further increase the reliability and robustness of SCR predictions.

Table 5: Estimated SCR error for insurer B based on an asymmetric approach (6) with one-sided thresholds.

$\tau = 0$, $\rho = 0$		β				
		1	**0.99**	**0.98**	**0.97**	**0.96**
α	**1**	-72.1	-56.0	-40.1	-30.2	-11.2
	0.98	-67.9	-55.4	-40.1	-30.2	-11.2
	0.96	-64.5	-51.2	-37.6	-29.1	-11.2
	0.94	-60.2	-47.1	-34.6	-22.4	-8.8
	0.92	-55.8	-41.9	-29.6	-16.3	-4.3
	0.9	-51.5	-36.3	-24.7	-13.1	2.4

Table 6: Estimated SCR error for insurer B based on an asymmetric approach (6).

$\tau = 0.02$, $\rho = 0$		β				
		1	**0.99**	**0.98**	**0.97**	**0.96**
α	**1**	-75.6	-60.1	-45.9	-32.0	-15.3
	0.98	-71.4	-59.7	-45.9	-32.0	-15.3
	0.96	-68.4	-56.6	-45.0	-32.2	-15.3
	0.94	-64.1	-51.2	-39.9	-29.9	-11.4
	0.92	-59.5	-46.6	-34.5	-23.8	-9.7
	0.9	-55.1	-42.0	-29.5	-16.4	-3.8

Table 7: Estimated SCR error for insurer B with the alternative setting and based on asymmetric approach (6).

$\tau = 0$, $\rho = 0$		β			
		1	**0.995**	**0.99**	**0.985**
α	**1**	-43.1	-43.1	-43.1	-43.1
	0.99	-37.0	-23.2	-7.1	-7.1
	0.98	-31.6	-18.4	-5.3	8.2
	0.97	-27.1	-12.0	-1.4	11.2
	0.96	-22.7	-7.3	3.1	14.0

Table 8: Estimated SCR error for insurer B with the alternative setting and based on asymmetric approach (6).

$\tau = 0.02$, $\rho = 0$		β			
		1	**0.995**	**0.99**	**0.985**
α	**1**	-58.3	-45.1	-45.1	-45.1
	0.99	-49.9	-36.7	-19.9	-19.9
	0.98	-44.7	-32.3	-18.0	-9.2
	0.97	-40.2	-28.1	-15.1	-6.2
	0.96	-36.0	-23.6	-11.8	-2.8

Table 9: Estimated SCR error for insurer C based on symmetric approach (5).

		β										
		1	**0.995**	**0.99**	**0.985**	**0.98**	**0.975**	**0.97**	**0.965**	**0.96**	**0.955**	**0.95**
α	**1**	21.7	22.5	19.8	17.7	17.4	15.8	15.0	14.2	14.5	14.0	13.9
	0.95	20.4	20.9	19.0	17.4	17.3	15.6	14.4	14.2	14.3	14.0	13.9
	0.9	19.7	20.1	18.4	17.1	16.9	15.6	14.3	13.4	13.7	14.3	14.6
	0.85	19.2	19.5	17.6	16.7	16.5	14.9	14.3	14.0	13.7	13.8	14.2
	0.8	18.9	19.2	17.3	16.7	16.2	15.4	13.9	14.0	13.5	14.6	14.0
	0.75	19.0	19.3	17.6	17.0	16.1	15.8	14.2	14.1	14.0	14.8	14.6
	0.7	19.2	19.5	17.9	17.4	16.4	16.0	14.7	14.3	14.9	14.8	15.1
	0.65	19.3	19.5	18.1	17.6	16.8	16.4	15.0	15.1	15.4	15.4	15.4
	0.6	19.6	19.8	18.4	18.0	17.1	16.9	15.4	15.6	15.7	15.9	15.9

Table 10: Estimated SCR error for insurer C based on an asymmetric approach (6).

β = 0.96		τ										
ρ = 0.06		**0.06**	**0.08**	**0.1**	**0.12**	**0.14**	**0.16**	**0.18**	**0.2**	**0.22**	**0.24**	**0.26**
α	**0.94**	22.0	22.5	23.4	25.6	27.0	28.8	30.8	32.8	35.8	38.1	39.7
	0.92	20.7	21.3	22.7	24.5	26.5	28.0	30.3	31.9	35.0	37.2	39.0
	0.9	18.7	19.6	21.3	23.1	25.6	27.1	28.7	30.6	33.2	36.2	37.8
	0.88	17.3	17.6	18.9	21.9	23.6	26.0	27.4	29.9	32.0	35.0	37.2
	0.86	15.2	16.1	17.2	18.8	22.1	24.2	26.3	27.9	30.2	32.8	35.8
	0.84	13.4	13.9	15.4	16.7	19.2	22.4	24.8	26.5	28.5	30.6	33.8
	0.82	9.6	9.8	11.8	14.6	16.9	19.2	22.3	24.9	26.5	28.7	31.2
	0.8	8.0	8.3	9.3	11.0	14.7	17.1	19.6	22.6	25.1	27.0	29.3
	0.78	3.7	4.3	6.2	8.0	10.4	14.3	16.8	19.8	22.8	25.1	27.0
	0.76	2.0	1.4	0.1	3.2	6.9	10.2	13.5	16.8	19.4	22.5	24.9

Bibliography

[1] van Beek, A. B. (2016). Continuity of Actuarial Models. Oldenburg: University Oldenburg.

[2] DAV-Arbeitsgruppe *Aggregationstechniken*. (2015). Proxy-Modelle für die Risikokapitalberechnung. Retrieved from https://aktuar.de/unsere-themen/fachgrundsaetze-oeffentlich/2015-07-08_DAV_Ergebnisbericht%20AG%20Aggregation.pdf

[3] F. Longstaff and E. Schwartz. (2001). Valuing American options by simulation: a simple least-squares approach. Review of Financial Studies, 14, 113-147.

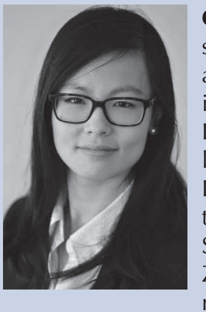
[4] Glasserman, P. (2003). Monte Carlo Methods in Financial Engineering. Springer.

[5] Jonen, C. (2011). Efficient Pricing of High-Dimensional American-Style Derivatives: A Robust Regression Monte Carlo Method. Cologne: University of Cologne.

[6] Koursaris, A. (2011). A Least Squares Monte Carlo Approach to Liability Proxy Modelling. Barrie&Hibbert.

Zoran Nikolić leitet die Abteilung Life/Health Modeling bei der Generali Deutschland AG. In dieser Funktion trägt er die Verantwortung für die Themen aktuarielle Bewertungsmodelle, ökonomische Szenarien und Proxy-Methoden. Vor 2010 hat Herr Nikolić in der Bilanzmathematik der Gothaer Leben gearbeitet. Seine Promotion hat er 2005 an der Universität Göttingen abgeschlossen. Herr Nikolić ist als Dozent bei der DAA und als Lehrbeauftragter an der Universität zu Köln tätig.

Christian Jonen ist seit April 2016 als Referent in der Abteilung Life/Health Calculation der Generali Deutschland AG tätig und für Solvency II Berechnungen im Rahmen des internen Modells verantwortlich. Zuvor war Herr Jonen viereinhalb Jahre bei HSBC im Bankwesen beschäftigt. Während seiner Promotion am Mathematischen Institut der Universität Köln galten seine Forschungsinteressen der Entwicklung von effizienten Monte-Carlo-Verfahren zur Bewertung von Finanzoptionen.



Chengjia Zhu ist seit Januar 2015 als Referentin in der Abteilung Life/Health Modeling der Generali Deutschland AG tätig. Nach dem Studium hat Frau Zhu als Unternehmensberaterin mehrere Jahre Versicherungsunternehmen in Bewertungs- und Modellierungsfragen unterstützt. Sie hat das Masterstudium „Mathematik und Anwendungsgebiete" an der Heinrich-Heine-Universität in Düsseldorf absolviert und ist seit 2015 Mitglied in der DAV.