MATHEMATISCHES INSTITUT
DER UNIVERSITÄT ZU KÖLN
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Cologne Computational Finance Laboratory

Summer 2012
April, 10th

Computational Finance - 1st Assignment

Deadline: April, 18th

Exercise 1 (Continuous interest rate) (oral exercise)

Assume a time horizon of one year ($t \in [0,1]$) and a discrete interest payment of $\frac{r}{m}$ at fixed dates $\frac{j}{m}$, $j \in \{1,\ldots,m\}$, $m \in \mathbb{N}$, $r \in [0,1]$. Then, investing an amount of K_0 at $t=0$ would result in a return of

$$K_1 = K_0 \cdot \left(1 + \frac{r}{m}\right)^m$$

at $t=1$. Explain the above formula, generalize it to any time horizon ($t \in [0,T]$) and derive a formula for continuous interest payments.

Exercise 2 (Hedging currency risks) (oral exercise)

An U.S. corporation will receive a fixed EUR–amount K at time $t=T$. For hedging its currency risk the corporation buys a currency forward, which offers a fixed EUR–USD exchange rate $F(0,T)$ (unit: [USD/EUR]) in $t=T$.

Assume risk free interest rates r_{USD} in the USD–zone and r_{EUR} in the EUR–zone and a today's USD–EUR exchange rate of $E(0)$ (unit: [USD/EUR]). What is the price of $F(0,T)$ at $t=0$?

Exercise 3 (Portfolios) (5 + 5 points)

In a financial market the following assets are traded: a share S as well as put and call options (all related to S) with three strike prices $K_1 \le K_2 \le K_3$.

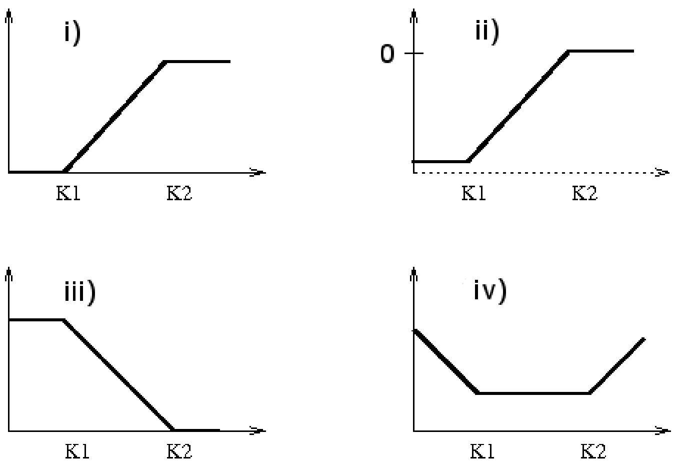
a) Sketch the payoffs of the following portfolios. What is the maximum profit or loss in each case?

   i) Put with strike K_1 as a short and as a long position respectively.

   ii) Call with strike K_1 as a short and as a long position respectively.

   iii) One call with strike K_1 (long), two calls with strike K_2 (short) and one call with strike K_3 (long).

b) For each of the following payoffs, construct portfolios out of vanilla options such that the payoff is met. (In example ii) note that the S-axis is shifted.)



Exercise 4 (No-Arbitrage Principle, Put-Call Parity) (3+7 points)

a) Use arbitrage arguments to prove the following bounds on option prices:

i) $V_{\mathrm{C}} \geq 0$

ii) $S \geq V_{\mathrm{C}}^{\mathrm{am}} \geq (S-K)^{+}$

b) Consider a portfolio consisting of three positions related to the same asset, namely one share (price S), one European put (value $V_{\mathrm{P}}^{\mathrm{eur}}$), plus a short position of one European call (value $V_{\mathrm{C}}^{\mathrm{eur}}$). Put and call have the same maturity date T, and no dividends are paid.

i) Show that the *put-call parity*

$$S + V_{\mathrm{P}}^{\mathrm{eur}} - V_{\mathrm{C}}^{\mathrm{eur}} = Ke^{-r(T-t)}$$

holds for all t, where K is the strike and r the risk-free interest rate.

ii) Use the put-call parity to show

$$V_{\mathrm{C}}^{\mathrm{eur}}(S,t) \geq S - Ke^{-r(T-t)},$$
$$V_{\mathrm{P}}^{\mathrm{eur}}(S,t) \geq Ke^{-r(T-t)} - S.$$

<u>Information</u>:

- The first exercise course takes place on April, 18th.
- Please prepare the oral exercises for presentation in the first exercise course and additionally hand in the written exercises there.
- Additional information can be found under

`www.mi.uni-koeln.de/~aschroet/vorlesung_ss_12` .