Localization in Lattice and Continuum Models of Reinforced Random Walks

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Abstract

We study the singular limit of a class of reinforced random walks on a lattice for which a complete analysis of the existence and stability of solutions is possible. We show that at a sufficiently high total density the global minimizer of a lattice 'energy' or Lyapunov functional corresponds to aggregation at one site. At lower values of the density the stable localized solution coexists with a stable spatially-uniform solution. Similar results apply in the continuum limit, where the singular limit of leads to a nonlinear diffusion equation. Numerical simulations of the lattice walk show a complicated coarsening process leading to the final aggregation.

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