Lyapunov functions and L^p-estimates for a class of reaction-diffusion systems

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Abstract

In this paper we give a sufficient condition for the existence of a Lyapunov function for the system

$$\left.\begin{array}{rcll} a_t & = & \nabla(k(a,c)\nabla a - h(a,c)\nabla c), & x\in\Omega, \quad t>0 \\ \varepsilon c_t & = & k_c\Delta c - f(c)c + g(a,c), & x\in\Omega, \quad t>0 \end{array}\right\} \quad (1)$$

for $\Omega \subset \mathbb{R}^N$ completed with either $a = c = 0$, or

$$\frac{\partial a}{\partial n} = \frac{\partial c}{\partial n} = 0, \text{ or } k(a,c)\frac{\partial a}{\partial n} = h(a,c)\frac{\partial c}{\partial n}, \ c = 0 \text{ on } \partial\Omega \times \{t > 0\}.$$

Furthermore we study the asymptotic behaviour of the solution and give some uniform L^p-estimates.

Keywords: Local existence of a solution, Lyapunov functions, asymptotic behaviour of the solution, uniform L^p-estimates
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