Lyapunov functions and $L^p$—estimates for a class of reaction-diffusion systems

Dirk Horstmann *

Abstract

In this paper we give a sufficient condition for the existence of a Lyapunov function for the system

\[
\begin{align*}
\frac{a_t}{\varepsilon c_t} &= \nabla (k(a,c)\nabla a - h(a,c)\nabla c), \quad x \in \Omega, \quad t > 0 \\
\frac{\varepsilon c_t}{c_t} &= k_c \Delta c - f(c) + g(a,c), \quad x \in \Omega, \quad t > 0
\end{align*}
\]

(1)

for $\Omega \subset \mathbb{R}^N$ completed with either $a = c = 0$, or

\[
\frac{\partial a}{\partial n} = \frac{\partial c}{\partial n} = 0, \text{ or } k(a,c)\frac{\partial a}{\partial n} = h(a,c)\frac{\partial c}{\partial n}, \quad c = 0 \text{ on } \partial \Omega \times \{t > 0\}.
\]

Furthermore we study the asymptotic behaviour of the solution and give some uniform $L^p$—estimates.

Keywords: Local existence of a solution, Lyapunov functions, asymptotic behaviour of the solution, uniform $L^p$—estimates

1991 MSC numbers: 35A07, 35B40, 35K40, 35K50, 35K55, 35K57

*Mathematisches Institut der Universität zu Köln, Weyertal 86 - 90, D-50931 Köln, Email address: dhorst@mi.uni-koeln.de