Liouville properties of covering spaces

Panagiotis Polymerakis

For a normal Riemannian covering $p: M \to N$ of a closed manifold $N$, we are interested in relations between the deck transformation group $\Gamma$ and the validity of the Liouville and the strong Liouville property on $M$. Our approach is probabilistic and relies heavily on the Brownian motion. Lyons and Sullivan constructed a discretization of the Brownian motion on $M$, obtaining a random walk on $\Gamma$. Their method was modified and extended in a work of Ballmann and Ledrappier, and in a recent joint work with Ballmann. In particular, it follows that there exists a probability measure $\mu$ on $\Gamma$ such that $H_b(M) \cong H_b(\Gamma, \mu)$ and $H^+(M) \cong H^+(\Gamma, \mu)$, where $H_b$ is the space of bounded harmonic functions and $H^+$ is the cone of positive harmonic functions. In this talk, we will survey some results that follow from this discretization. Finally, we will show that if $\Gamma$ is of exponential growth, then $M$ does not have the strong Liouville property, which was conjectured by Lyons and Sullivan.