Liouville properties of covering spaces

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For a normal Riemannian covering $p\colon M \to N$ of a closed manifold N, we are interested in relations between the deck transformation group Γ and the validity of the Liouville and the strong Liouville property on M. Our approach is probabilistic and relies heavily on the Brownian motion. Lyons and Sullivan constructed a discretization of the Brownian motion on M, obtaining a random walk on Γ. Their method was modified and extended in a work of Ballmann and Ledrappier, and in a recent joint work with Ballmann. In particular, it follows that there exists a probability measure μ on Γ such that $H_b(M) \cong H_b(\Gamma, \mu)$ and $H^+(M) \cong H^+(\Gamma, \mu)$, where H_b is the space of bounded harmonic functions and H^+ is the cone of positive harmonic functions. In this talk, we will survey some results that follow from this discretization. Finally, we will show that if Γ is of exponential growth, then M does not have the strong Liouville property, which was conjectured by Lyons and Sullivan.