SS 2022

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Geometrische Topologie

Übungsblatt 8

Aufgabe 1. Consider the map

$$\begin{array}{rccl} f\colon & \mathbb{C}^* & \longrightarrow & \mathbb{C} \\ & z & \longmapsto & 2(z+1/z) \end{array}$$

(a) Show that f is a branched covering. Determine the branch points and the branching indices of their preimages.

(b) Determine the image of the restriction of this map to the annulus $\{z \in \mathbb{C}\colon\ 1/2 < |z| < 2\}$.

Aufgabe 2. Show that for $g, h \geq 2$ there is an (unbranched) covering $\Sigma_g \to \Sigma_h$ if and only if $h-1$ divides $g-1$.

Aufgabe 3. (a) Give a topological description of a 3-fold branched covering

$$\text{annulus} \longrightarrow D^2$$

with three branch points in D^2.

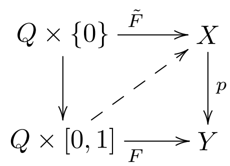
Hint: Describe D^2 as the gluing of three pairs of neighbouring edges of an enneagon, the annulus as the suitable gluing of three copies of this enneagon.

(b) Verify the Riemann–Hurwitz formula for this example.

Aufgabe 4. Let $p\colon X \to Y$ be a covering.

(a) Let α be a **path** in Y, i.e. a continuous map $\alpha\colon [0,1] \to Y$. Let $y_0 = \alpha(0)$. Choose a point $x_0 \in p^{-1}(y_0)$. Show that there is a unique path $\tilde{\alpha}$ in X with $\tilde{\alpha}(0) = x_0$ and $p \circ \tilde{\alpha} = \alpha$.

(b) Let Q be a connected topological space and $F\colon Q \times [0,1] \to Y$ a continuous map, which we regard as a homotopy of maps $Q \to Y$. Assume there is a continuous map $\tilde{F}\colon Q \times \{0\} \to Y$ with $p \circ \tilde{F} = F|_{Q\times\{0\}}$. Show that there is a continuous map $\tilde{F}: Q \times [0,1] \to Y$ that coincides on $Q \times \{0\}$ with the given one and satisfies $p \circ \tilde{F} = F$. In other words, one can complete the following commutative diagram:



This is called the **homotopy lifting property**.

(c) Show that for X path connected and Y simply connected, the covering p must be a homeomorphism.

Remark: A covering $\tilde{Y} \to Y$ with $\tilde{Y}$ path connected, locally path connected and simply connected is called **universal covering**. Part (c) shows the uniqueness of the universal covering.

Abgabe: Mittwoch 1.6.22
bis spätestens 17:00 Uhr im Raum 206/7 des MI