HARMONIC ALMOST-COMPLEX STRUCTURES ON TWISTOR SPACES

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If an even-dimensional Riemannian manifold (N, h) admits an almost-Hermitian structure, it has many and it is natural to seek for "reasonable" criteria that distinguish some of these structures. A natural way to obtain such criteria is to consider the almost-Hermitian structures on (N, h) as sections of the twistor bundle $\mathcal{T}$. Motivated by the harmonic maps theory, C.Wood has suggested to consider as "optimal" the almost-Hermitian structures $J : (N, h) \rightarrow (\mathcal{T}, \tilde{h})$ which are critical points of the energy functional under variations through sections of $\mathcal{T}$ where $\tilde{h}$ is the natural Riemannian metric on $\mathcal{T}$ induced by h and the standard metric of the fibre. These critical points are not harmonic maps in general but, by analogy, they are referred to as "harmonic almost-complex structures". The main result of this talk states that the Atiyah-Hitchin-Singer and Eells-Salamon almost-complex structures on the negative twistor space of an oriented Riemannian four-manifold N (i.e. the component of $\mathcal{T}$ whose sections are the almost-Hermitian structures compatible with the opposite orientation of N) are harmonic in the sense of C.Wood if and only if the base manifold is, respectively, self-dual or self-dual and of constant scalar curvature. The stability of these almost-complex structures will be also discussed.

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