# MPS formulation of quasi-particle wave functions

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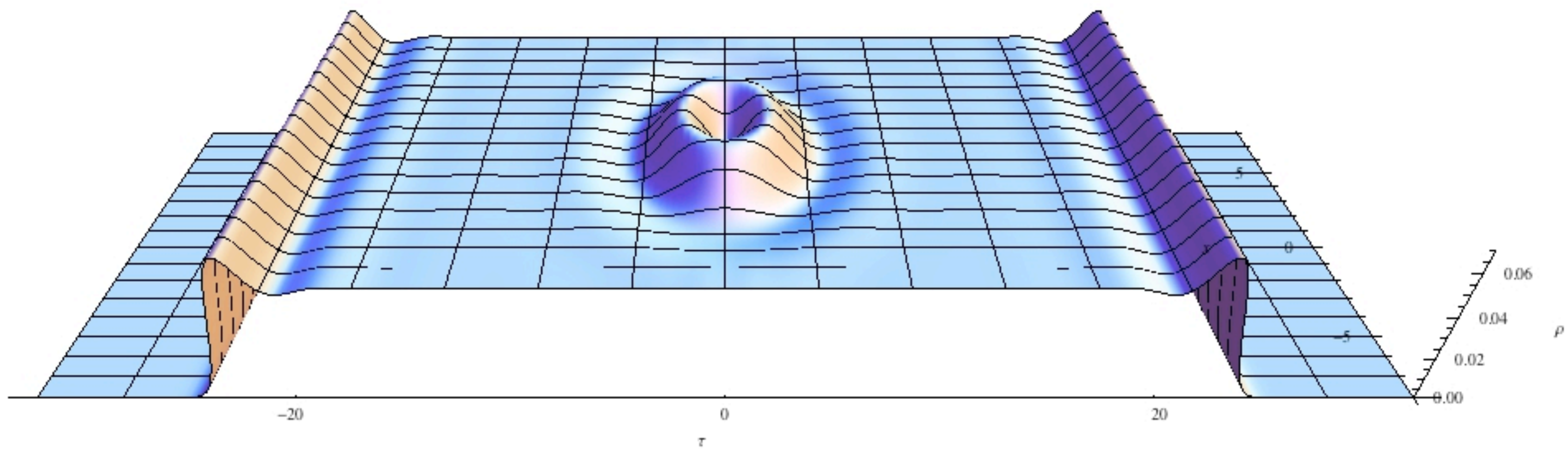
# Outline

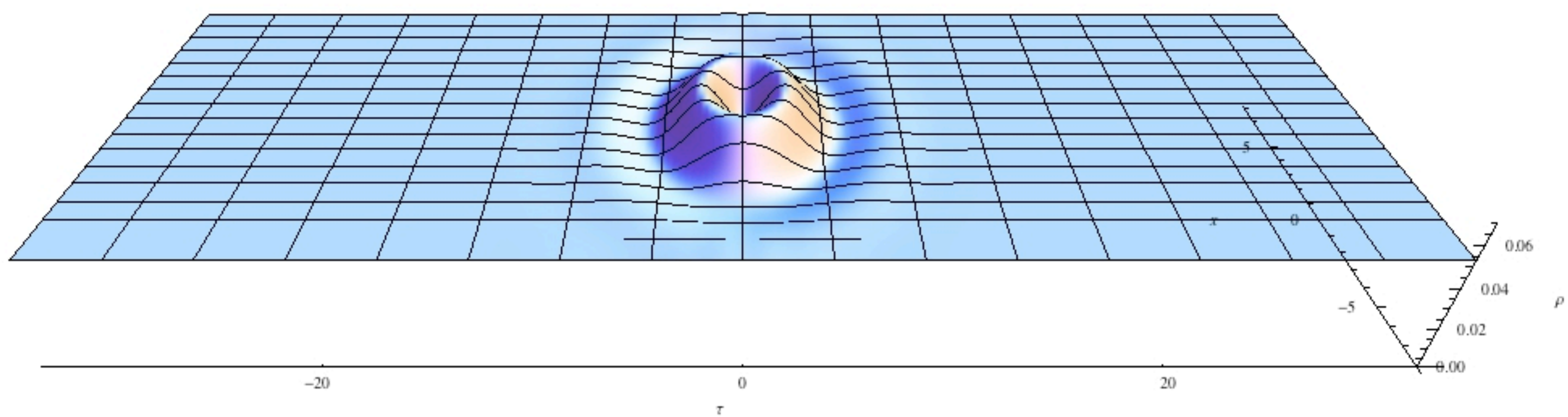
- ⋆ Short review of matrix product states
- ⋆ Quasi-particles in the quantum hall effect
- ⋆ MPS for quantum hall states
- ⋆ MPS for quasi-particles
- ⋆ Outlook

# Result

MPS description for quasi-particles in the Laughlin state.
$\nu = 1/3$, $L_x = 16$, $P_{max} = 8$, $N_e = 40$, finite case:




Infinite case:



# Matrix product states

MPS: tailor made to describe states with a finite amount of entanglement, such as symmetry protected topological phases.

A general state can represented as

$$|\Psi\rangle = \sum_{\{p_i\}} \sum_{a_2,\ldots,a_N} M^{[p_1]}_{a_1,a_2} M^{[p_2]}_{a_2,a_3} \cdots M^{[p_N]}_{a_N,a_{N+1}} |p_1, p_2, \ldots, p_N\rangle$$

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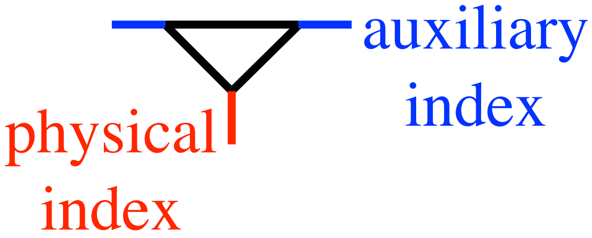
The matrices: $M^{[p_i]}_{a_i,a_{i+1}}$



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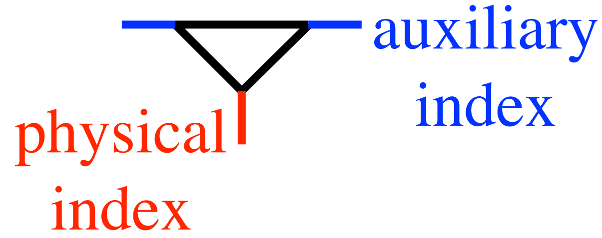
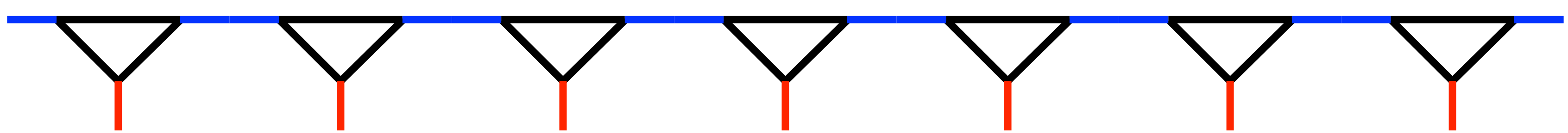



A (coefficient of a) state:



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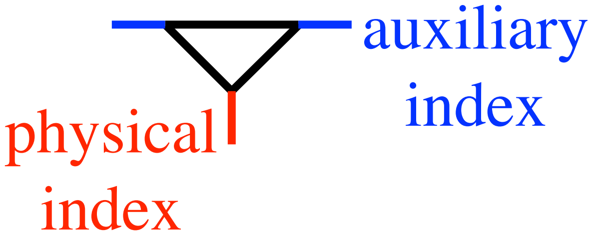


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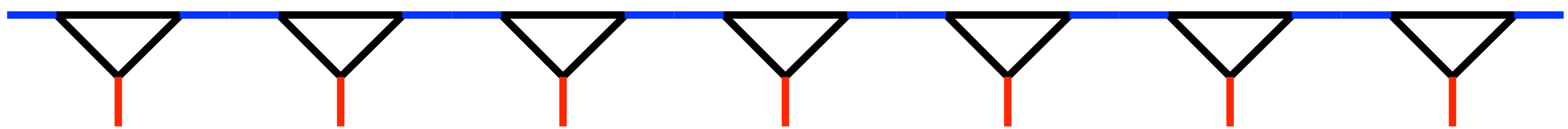


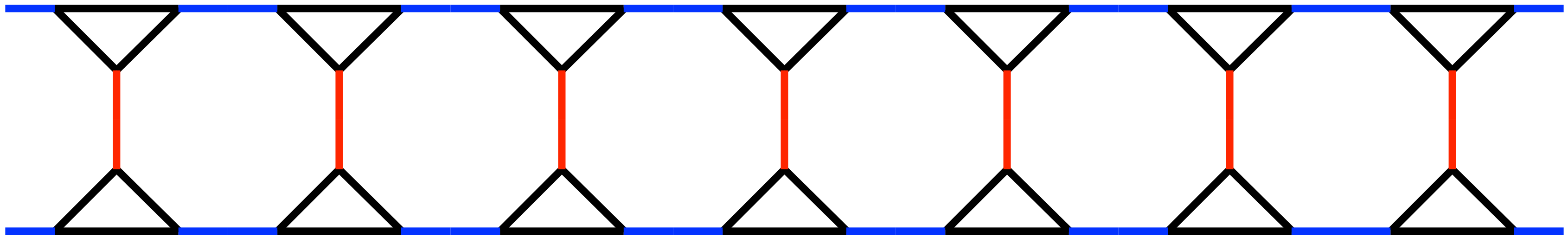
The auxiliary index takes the values: $a_i = 1, 2, \ldots, d$

For many interesting states, a low value of *d* suffices!

# Matrix product states

MPS allow for efficient calculation of expectation values, without having to know the state *(exponential # of coefficients)* explicitly.

The norm of a state:



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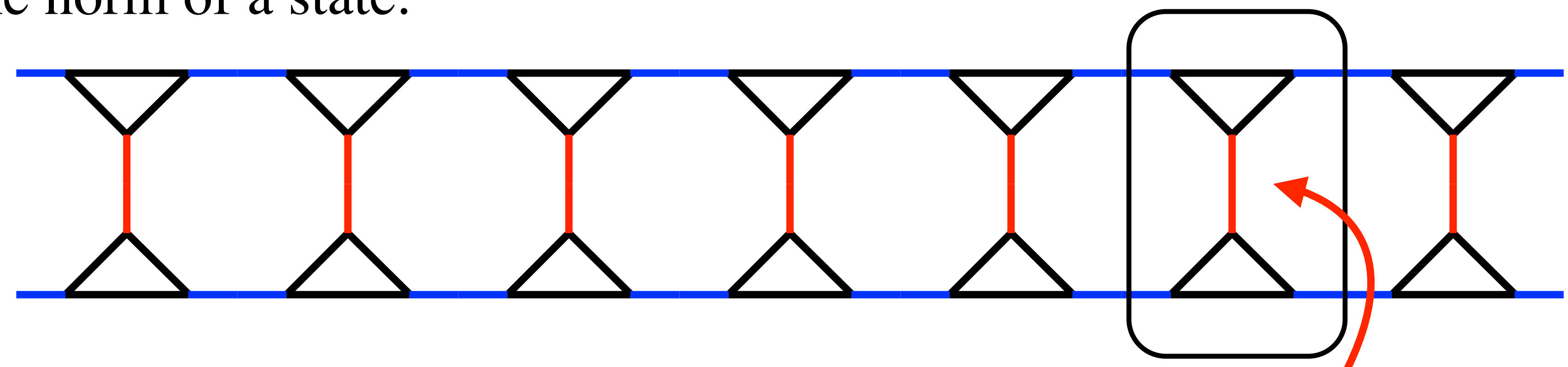

Contract the physical indices first!

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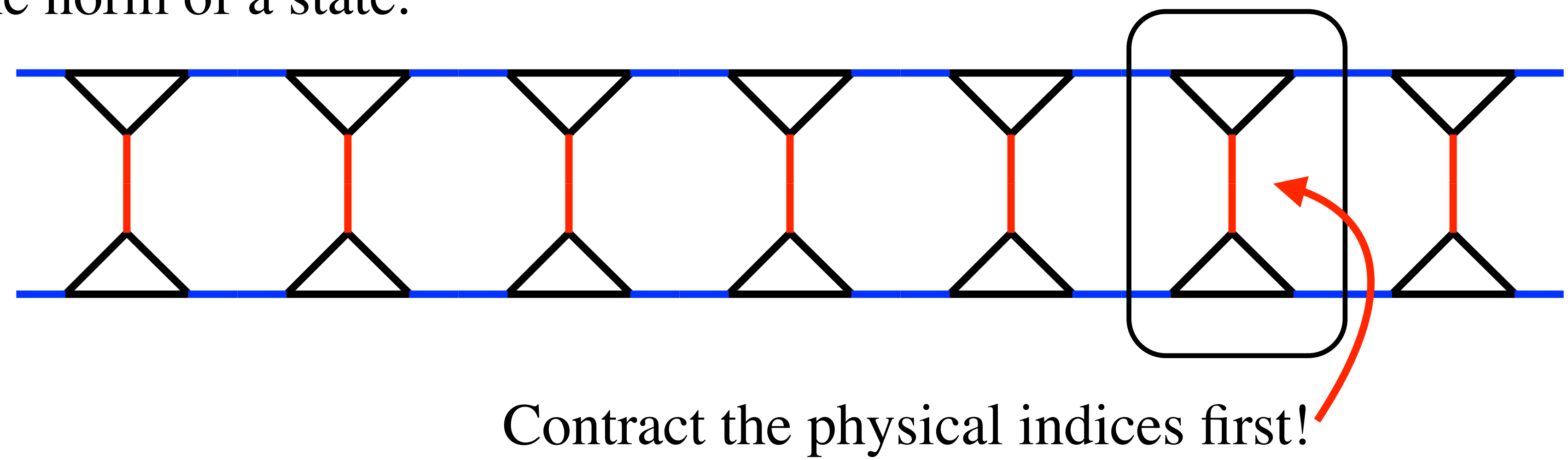
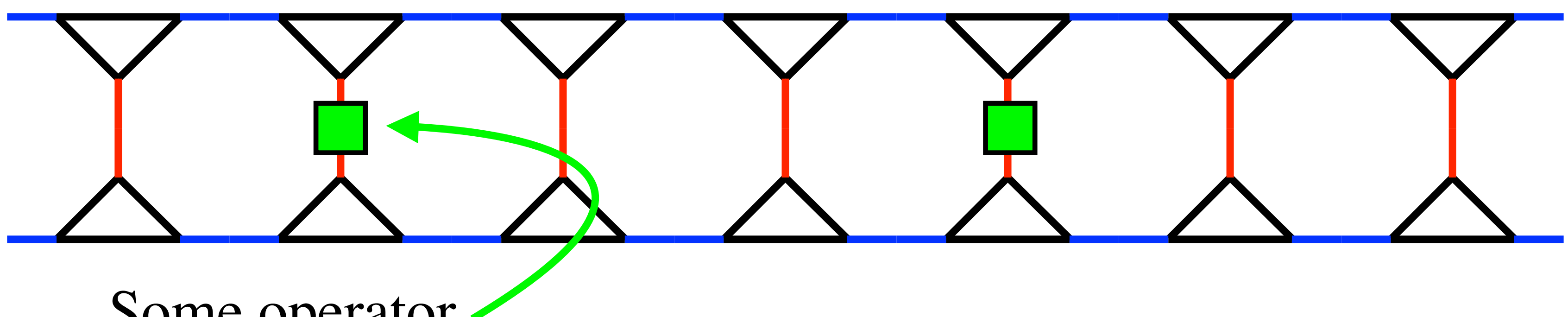



Correlation function/density:



# Matrix product states

Prototypical example:
AKLT state, Haldane gap in the spin-1 Heisenberg anti-ferromagnet.

Spin-1 Heisenberg hamiltonian: $H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$

The AKLT hamiltonian

$$H_{\mathrm{AKLT}} = \sum_j P^{(2)}_{j,j+1} = \sum_j \frac{1}{2} \vec{S}_j \cdot \vec{S}_{j+1} + \frac{1}{6} \left( \vec{S}_j \cdot \vec{S}_{j+1} \right)^2 + \frac{1}{3} \mathbf{1}_j$$

In the ground state, no neighbouring two spin-1's are in the spin-2 channel!

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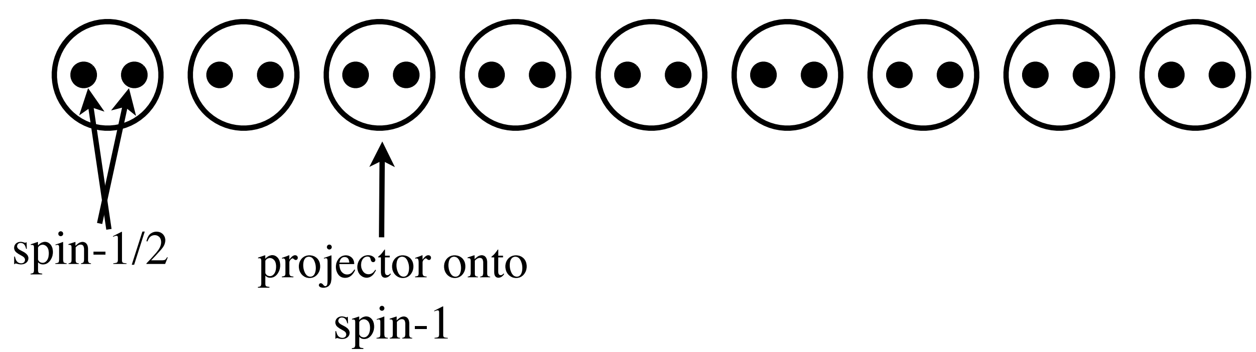
'Caricature' of the ground state:



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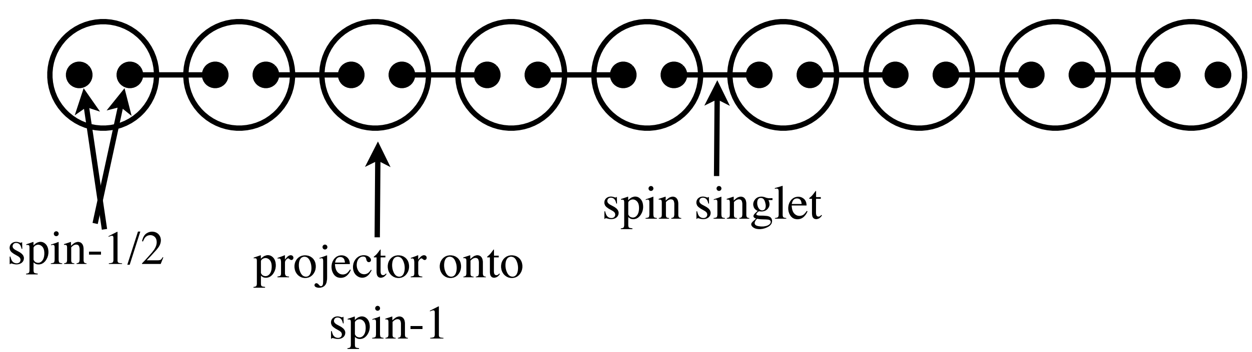

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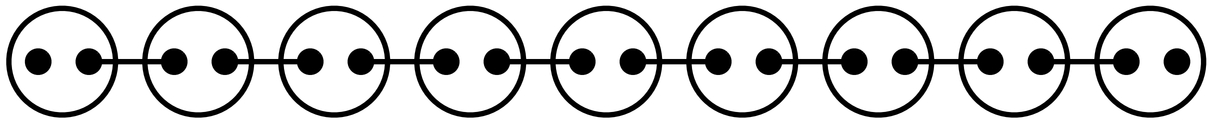

This is an MPS with bond dimension two.

Description of the singlets in the space of spin-1/2's: $A = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$$|\Psi_{\text{singlets}}\rangle = \sum_{\{l,r\}} A_{r_1,l_2} A_{r_2,l_3} \cdots A_{r_{N-1},l_N} |l_1, r_1, l_2, r_2, \ldots, l_N, r_N\rangle$$

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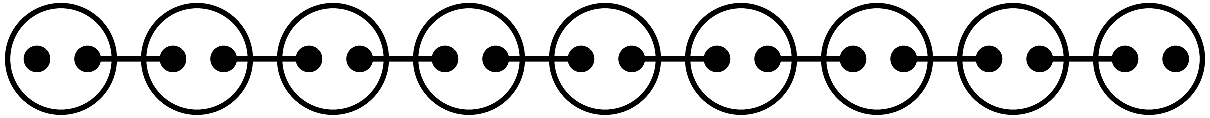

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Projectors onto spin-1: $P^+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad P^0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad P^- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

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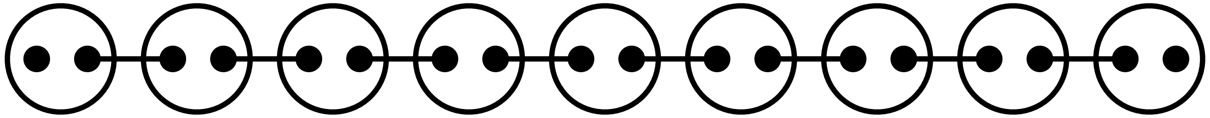

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MPS formulation of the AKLT state: $M^{[\sigma]} = P^\sigma A$

$$|\Psi_{\text{AKLT}}\rangle = \sum_{\{\sigma_i=+,0,-\}} M^{[\sigma_1]} M^{[\sigma_2]} \ldots M^{[\sigma_N]} |\sigma_1, \sigma_2, \ldots, \sigma_N\rangle$$

# Why MPS for qH quasi-***particles***?

Model states, such as Laughlin & Moore-Read are *ground states* of model hamiltonians, and can be written as a *single CFT correlator*

Quasi-***holes*** in these state can also be described in terms of a single CFT correlator.
Conjecture: braiding phase given by the *monodromy* of the correlator.

Quasi-***particles*** are harder to describe, and fewer analytic tools are available.

An MPS description would give access to large enough system sizes to numerically determine the braid factors.

The interesting braid factor: quasi-hole around a quasi-particle (opposite sign in comparison to two quasi-holes or two quasi-particles).

# Quantum Hall wave functions

The Laughlin state: $\Psi_{\mathrm{L}} = \prod_{i<j} (z_i - z_j)^m \mathcal{G}_{\mathrm{exp}}$

Dependence on the geometry is hidden in $\mathcal{G}_{\mathrm{exp}}$

$$\prod_{i<j} (z_i - z_j)^m = \sum_{\lambda} c_{\lambda} \mathrm{sl}_{\lambda}(\{z_i\})$$

Finding all the $c_\lambda$ is a hard problem, but can be formulated as an MPS

# qH wave functions as CFT correlators

To obtain an MSP description, one starts with CFT correlators:

$$\prod_{i<j}(z_i - z_j)^m = \langle V(z_1)V(z_2)\cdots V(z_N)\mathcal{O}_{\rm bg}\rangle$$

$$V(z) =: e^{i\sqrt{m}\phi(z)} : \qquad H(w) =: e^{i/\sqrt{m}\phi(w)} :$$

In the presence of a quasi-hole:

$$\prod_{i<j}(z_i - z_j)^m \prod_k (z_k - w) = \langle H(w)V(z_1)V(z_2)\cdots V(z_N)\mathcal{O}_{\rm bg}\rangle$$

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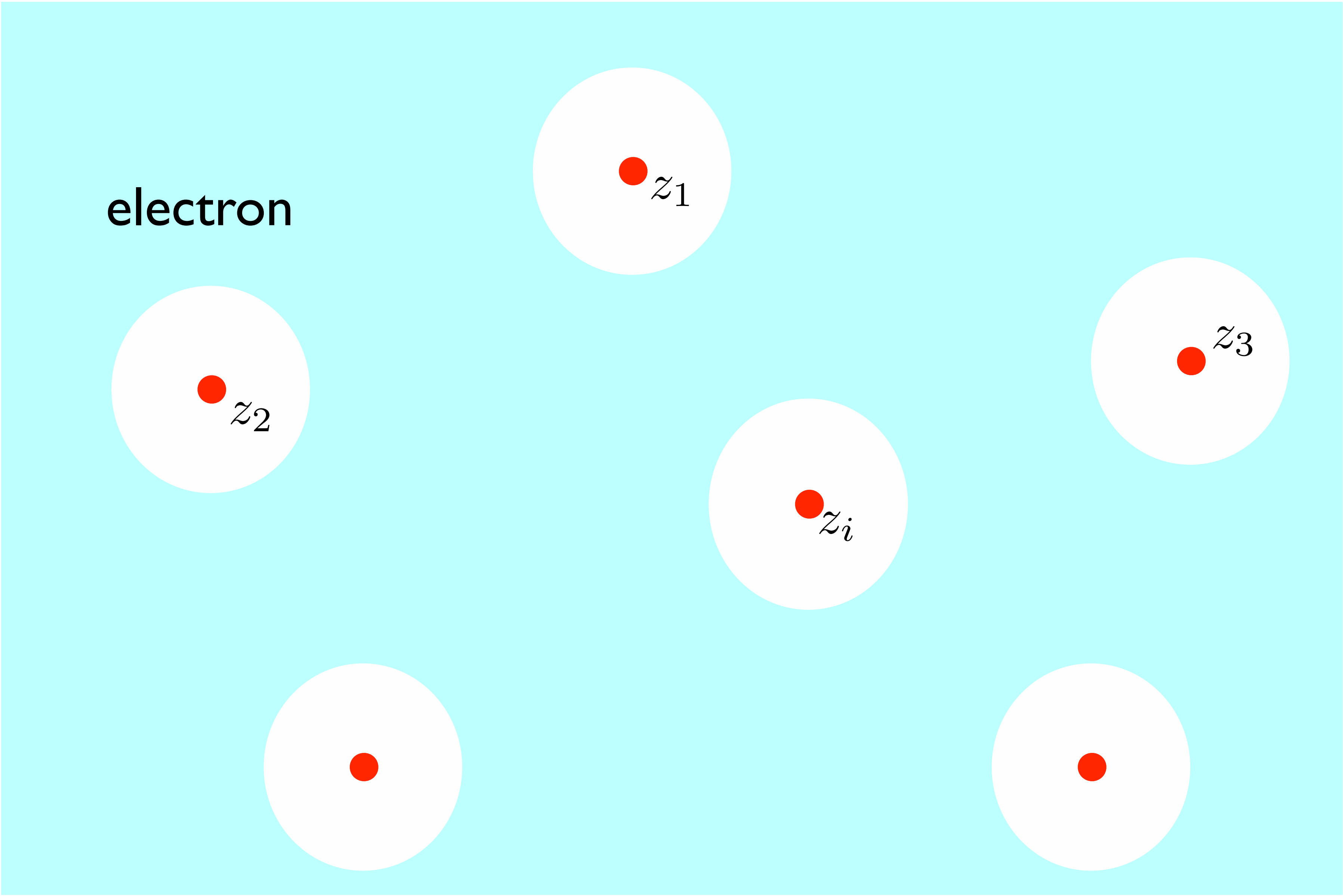
The naive inverse quasi-hole does **not** give a **physical** quasi-particle because of the poles:

$$\prod_{i<j}(z_i - z_j)^m \prod_k (z_k - \eta)^{-1} = \langle H^{-1}(\eta)V(z_1)V(z_2)\cdots V(z_N)\mathcal{O}_{\mathrm{bg}}\rangle$$

# The idea: qH quasi-***particles***

Hansson et al.



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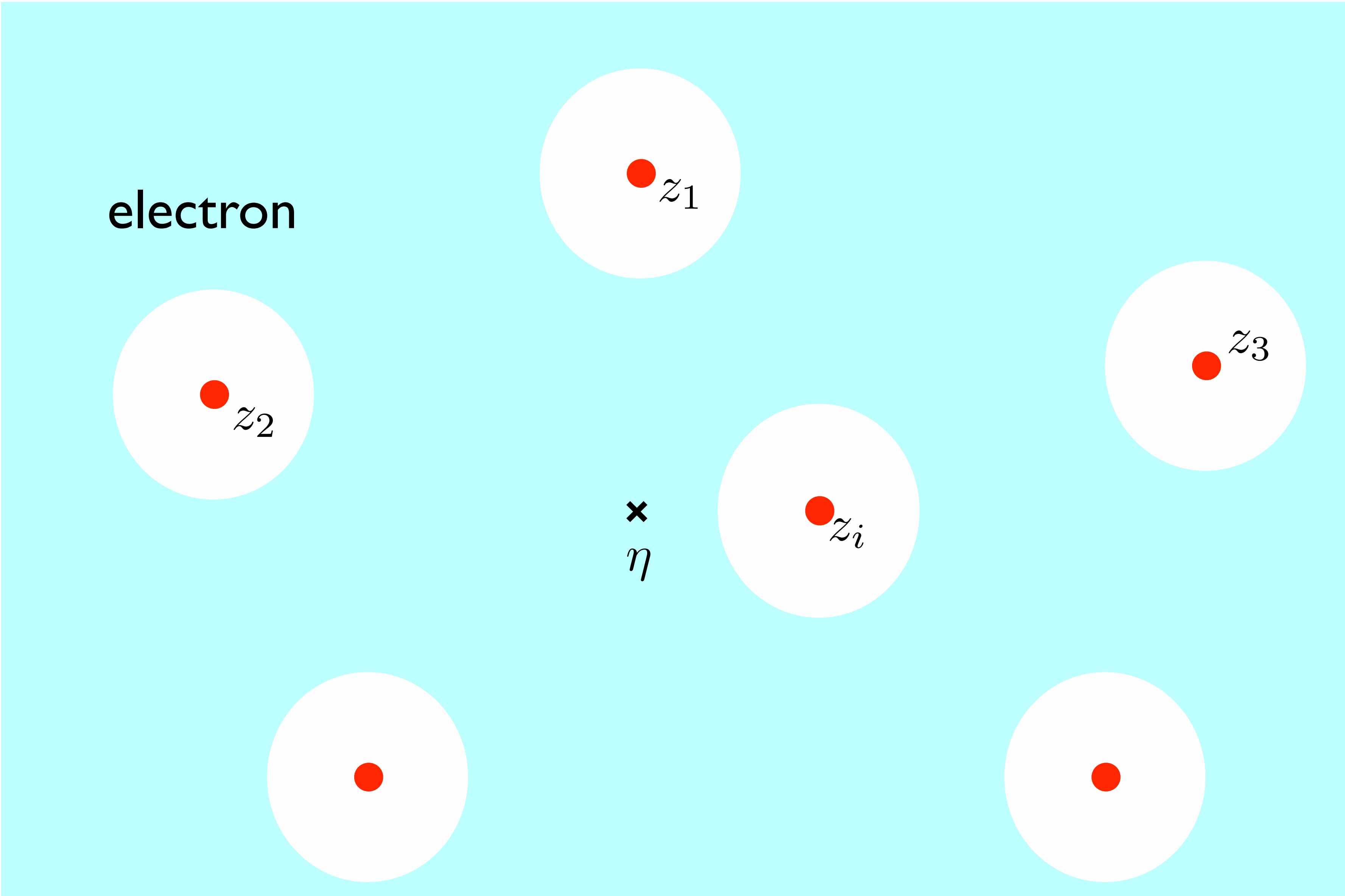

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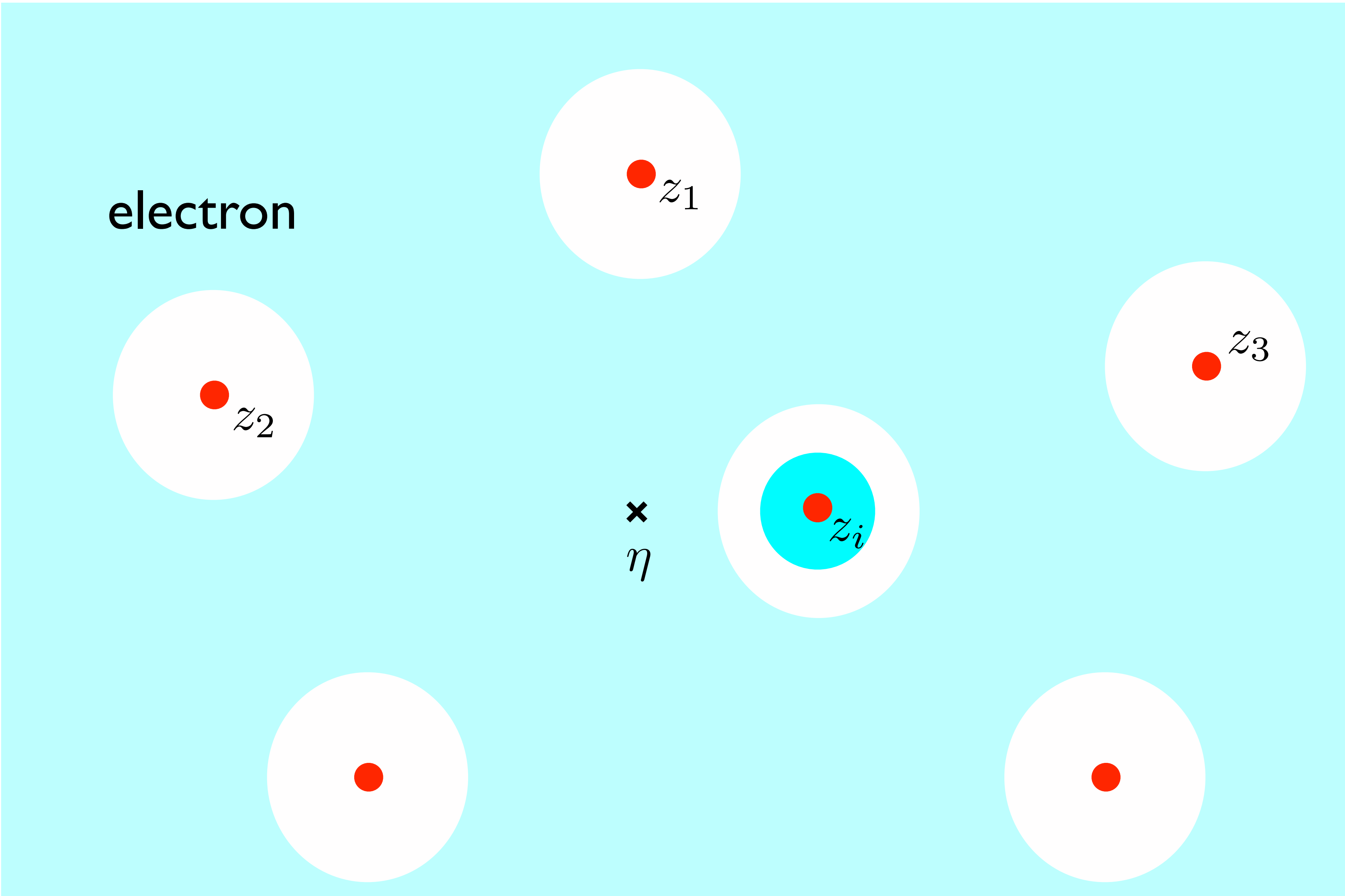

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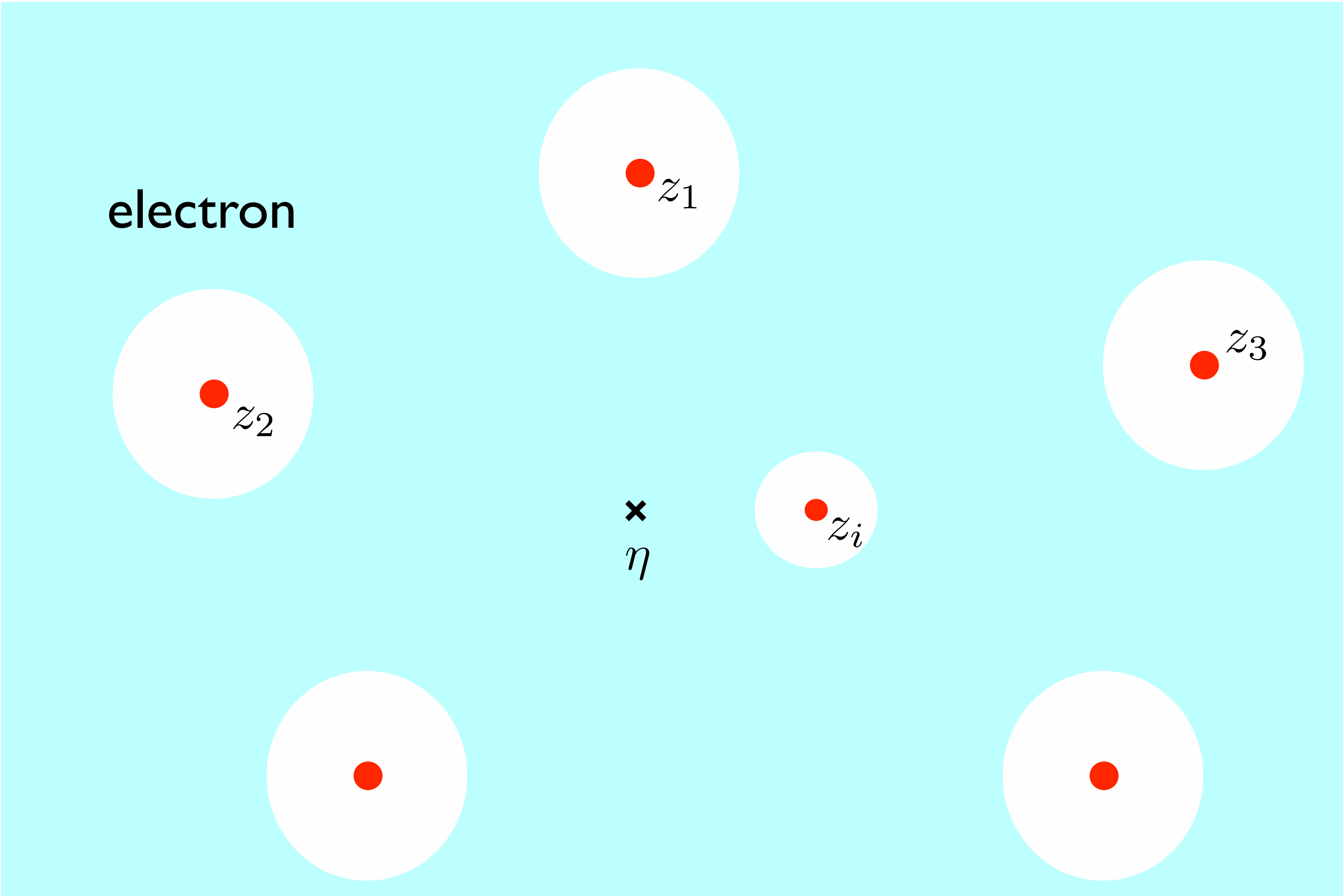

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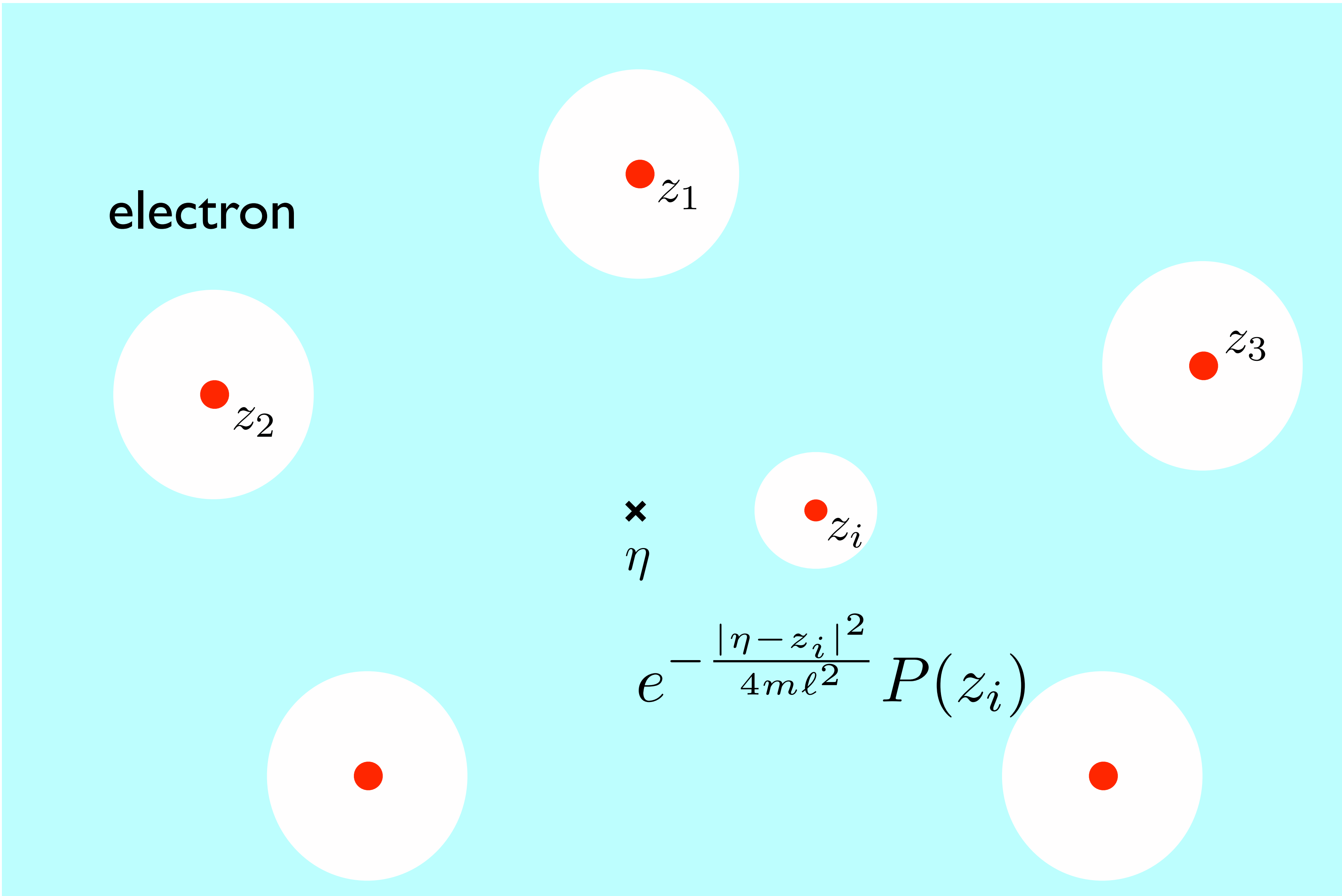

# Delocalized 'holes' and 'particles'

Expanding the quasi-hole wave function gives a set of delocalized, angular momentum 'quasi-hole' states $\Psi^{l}_{\mathrm{qh}}$ :

$$\prod_{i<j}(z_i - z_j)^m \prod_k (z_k - w) =$$
$$w^{N_e}\Psi^{l=0}_{\mathrm{qh}} + w^{N_e-1}\Psi^{l=1}_{\mathrm{qh}} + \cdots + w^{1}\Psi^{l=N_e-1}_{\mathrm{qh}} + w^{0}\Psi^{l=N_e}_{\mathrm{qh}}$$

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The modified electron operator gives a set of delocalized, angular momentum 'quasi-***particle***' states $\Psi^l_{\rm qp}$ (equal to Jain's quasi-particle):

$$\Psi^l_{\rm qp} = \sum_i (-1)^i z_i^l \prod_{j<k}^{(i)} (z_j - z_k)^m \partial_i \prod_l^{(i)} (z_l - z_i)^{m-1} =$$
$$= \sum_i (-1)^i z_i^l \langle V(z_1)\cdots V(z_{i-1}) P(z_i) V(z_{i+1}) \cdots V(z_{N_e}) \mathcal{O}_{\rm bg}\rangle$$

$$P(z) = \partial : e^{i(\sqrt{m} - 1/\sqrt{m})\phi(z)} :$$

# Localized quasi-particle states

Localizing a quasi-particle on the disk: the localizing exponential factor is the projector onto the lowest Landau level

$$\mathcal{P}_{\mathrm{LLL}} = \sum_j \psi_j^*(\eta)\psi_j(z) \propto e^{-\frac{1}{4ml_b^2}|z-\eta|^2}$$

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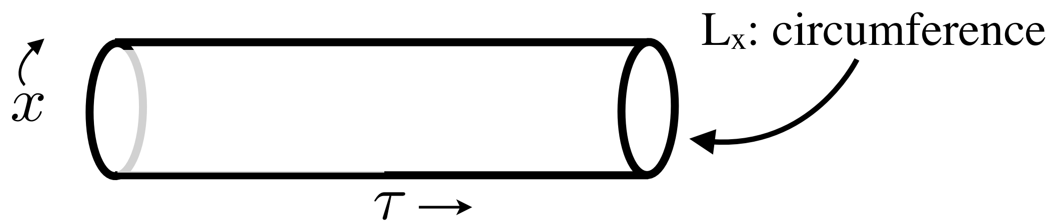


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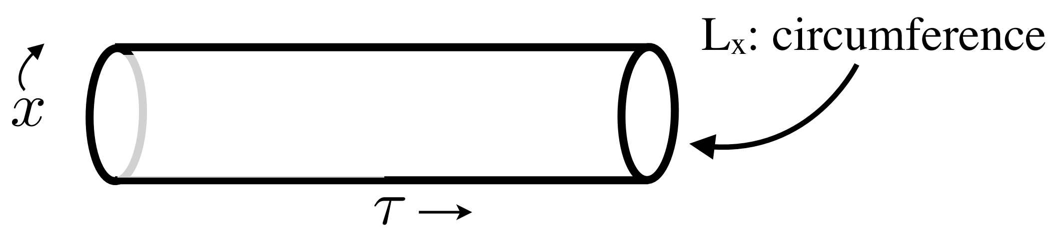


$$z = x + i\tau$$

$$\psi_k(x,\tau) = \frac{1}{\sqrt{L_x l_b \sqrt{\pi}}} e^{-\frac{1}{l_b^2}\left(ix\tau_k + (\tau-\tau_k)^2/2\right)} \qquad \tau_k = \frac{2\pi l_b^2 k}{L_x}$$

$$\mathcal{P}_{\mathrm{LLL}} = \sum_k \psi_k^*(x_\eta, \tau_\eta)\psi_k(x,\tau) \propto \sum_n e^{-\frac{1}{4ml_b^2}|z_\eta - z + L_x n|^2}$$

# MPS for the quantum Hall effect

Dubail et al., **Zaletel & Mong**, Princeton/Paris group

From the CFT correlation function, we obtain an MPS expression:

$$\sum_\lambda c_\lambda \mathrm{sl}_\lambda(\{z_i\}) = \langle V(z_1)V(z_2)\cdots V(z_N)\mathcal{O}_{\mathrm{bg}}\rangle$$

$$= \sum_{\alpha_i} \langle \mathrm{out}|V(z_1)|\alpha_1\rangle\langle\alpha_1|V(z_2)|\alpha_2\rangle\cdots\langle\alpha_{N_e-1}|V(z_N)|\mathrm{in}\rangle$$

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$c_\lambda$: obtained by using the mode expansion of the vertex operators:

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To obtain $M^{[0]}$ and $M^{[1]}$: use the commutators of the free boson CFT, f.i.

$$M_j^{[1]} = \frac{1}{2\pi i}\oint \frac{dz}{z} z^{-j}\langle\alpha|V(z)|\alpha'\rangle = \langle\alpha|V_{-j-h}|\alpha'\rangle$$

# Remarks about some details

The auxiliary space is the space of states in the free boson CFT:

$$\phi(z) = \phi_0 + i a_0 \ln(z) + i \sum_{n \neq 0} \frac{1}{n} a_n z^{-n}$$

$$[a_n, a_m] = n\delta_{n+m,0} \qquad [\phi_0, a_0] = i \qquad a_n^\dagger = a_{-n}$$

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The auxiliary space is truncated by a finite momentum:

$$P = \sum_j j m_j \leq P_{\max}$$

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$$\phi(z) = \phi_0 + i a_0 \ln(z) + i \sum_{n \neq 0} \frac{1}{n} a_n z^{-n}$$

$$[a_n, a_m] = n\delta_{n+m,0} \qquad [\phi_0, a_0] = i \qquad a_n^\dagger = a_{-n}$$

The charge Q and occupation numbers of the neutral modes, $m_i$ are the relevant labels: $|Q; m_1, m_2, \ldots\rangle$

The auxiliary space is truncated by a finite momentum:

$$P = \sum_j j m_j \leq P_{\max}$$

The background charge is taken care of by insertion of charge 1/m on every orbital, via $e^{-i\phi_0/\sqrt{m}}$. The M[1] become *site independent*!

$$V_{-n-h} = e^{in/\sqrt{m}\phi_0} V_{-h} e^{-in/\sqrt{m}\phi_0}$$

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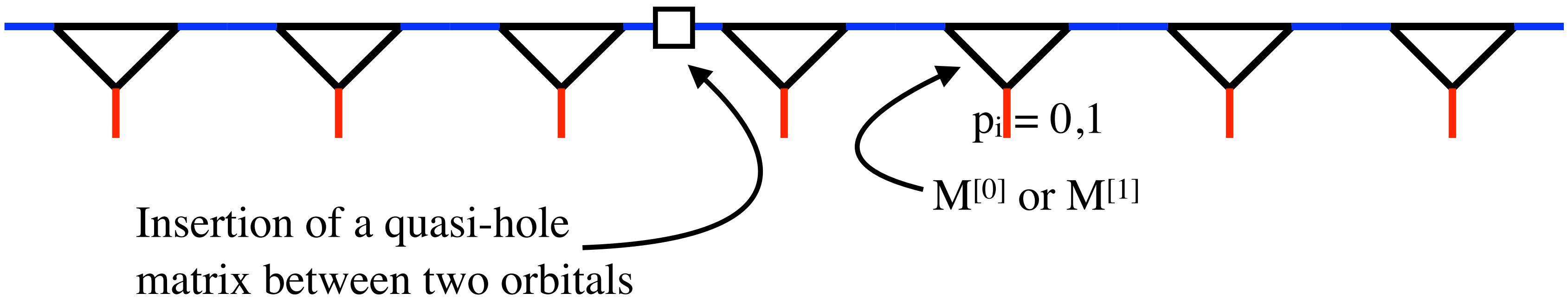
The correct cylinder normalization is obtained from the free time evolution along the cylinder (other geometries: normalize by hand).

# MPS for quasi-hole states

Following Zaletel & Mong, we can write the Laughlin state with quasi-holes as follows:

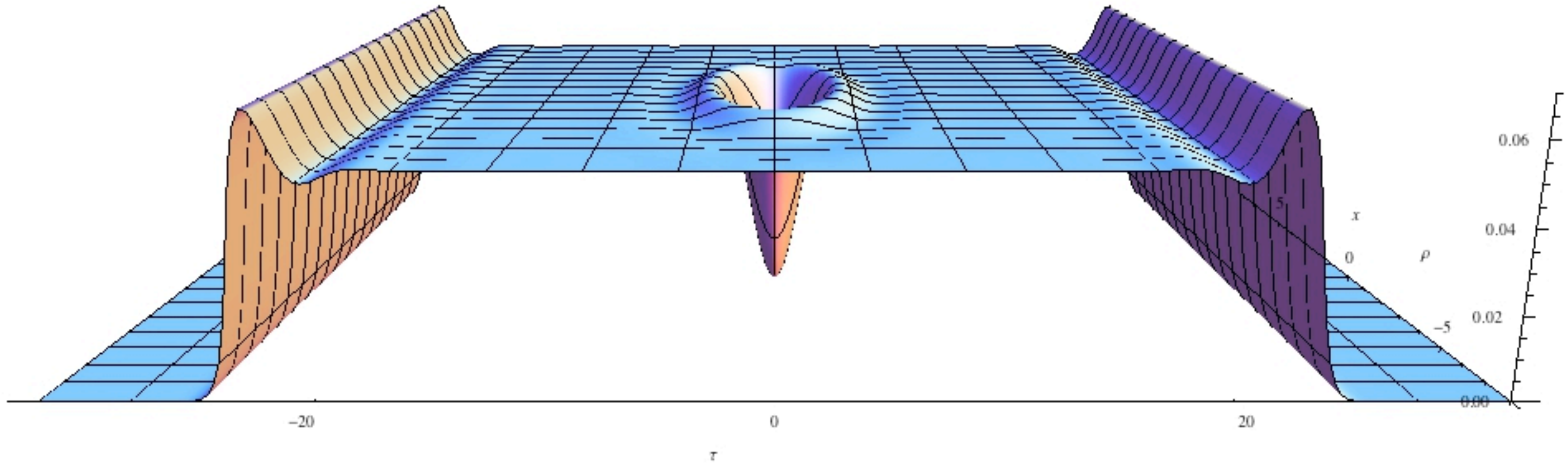


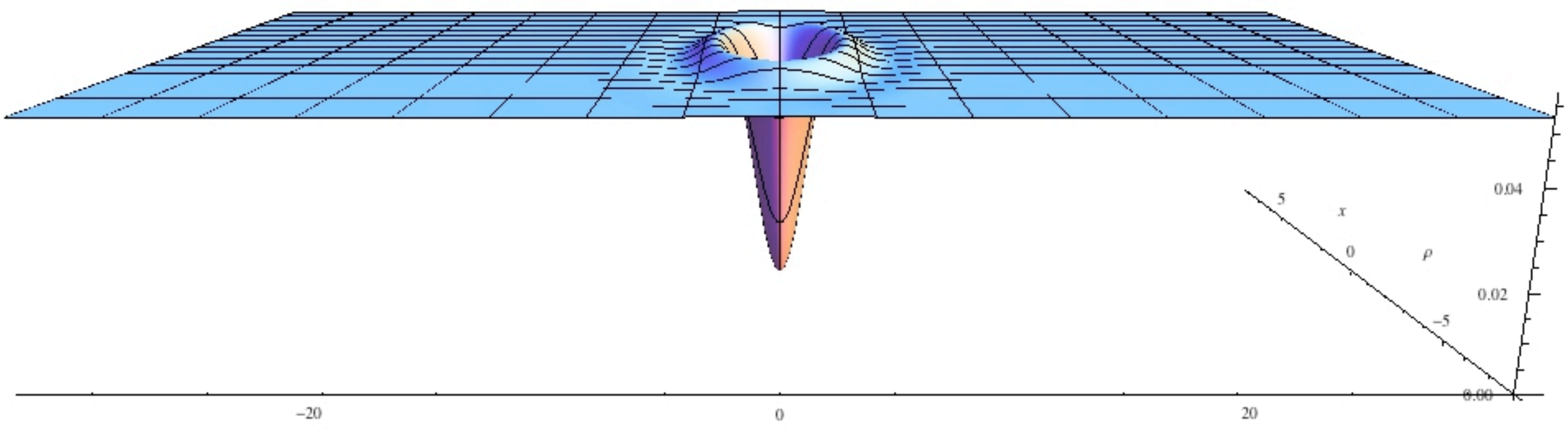

This gives the coefficients $c_\lambda$, which are checked by expanding the wave functions for a small number of particles explicitly.

The resulting state does not depend on where we insert the the quasi-hole matrix (for large enough $P_{max}$).

# MPS for quasi-hole states

Density of 1/3 state, with $P_{max} = 8$, $L_x = 16$, for 40 electrons:

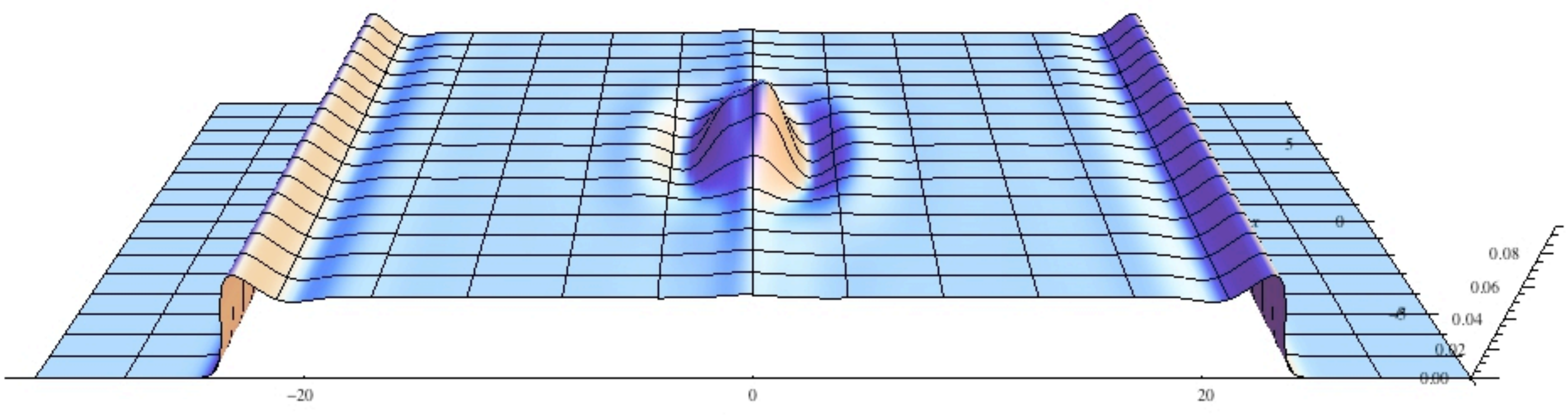


# MPS for naive quasi-particle state

If we use the naive (wrong!) quasi-particle, we do not get a nicely localized charge.

The density strongly depends on where we insert the quasi-particle operator!

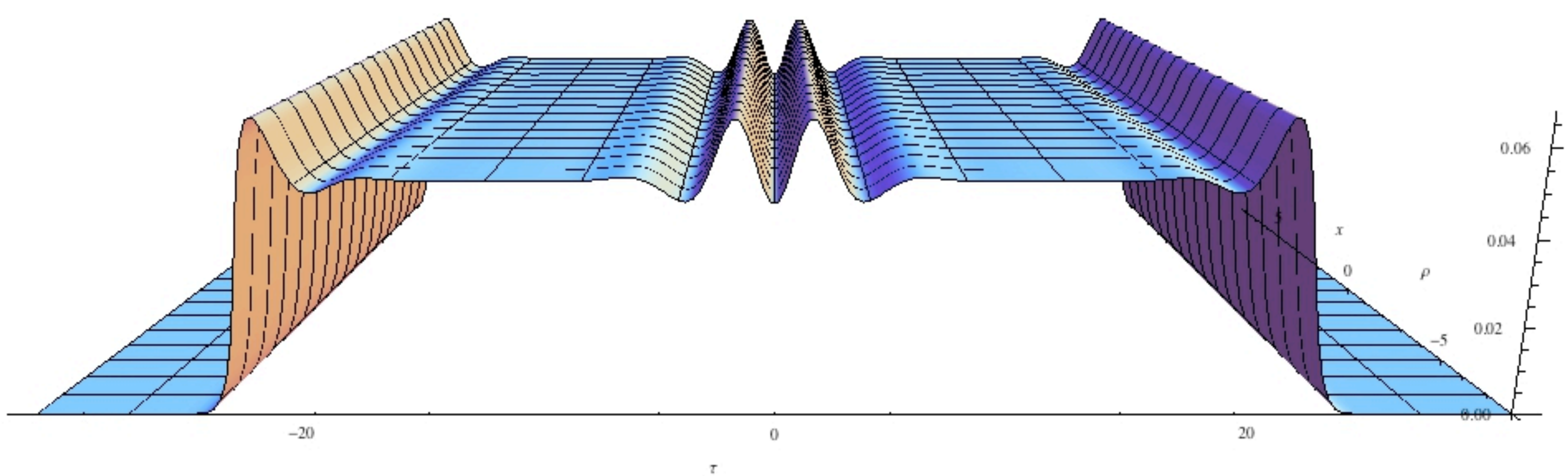


# MPS for delocalized 'quasi-particle'

To obtain a good quasi-particle, we first construct the delocalized quasi-particle states (angular momentum states).

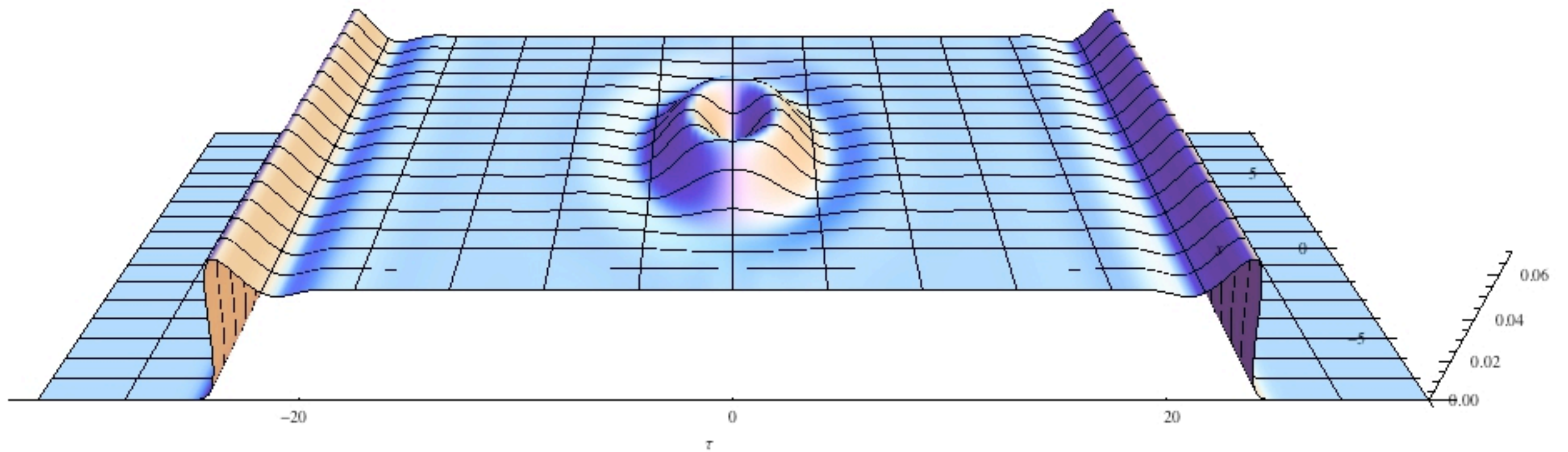
$$\Psi_{\mathrm{qp}}^{l} = \sum_i (-1)^i z_i^l \langle V(z_1) \cdots V(z_{i-1}) P(z_i) V(z_{i+1}) \cdots V(z_{N_e}) \mathcal{O}_{\mathrm{bg}} \rangle$$

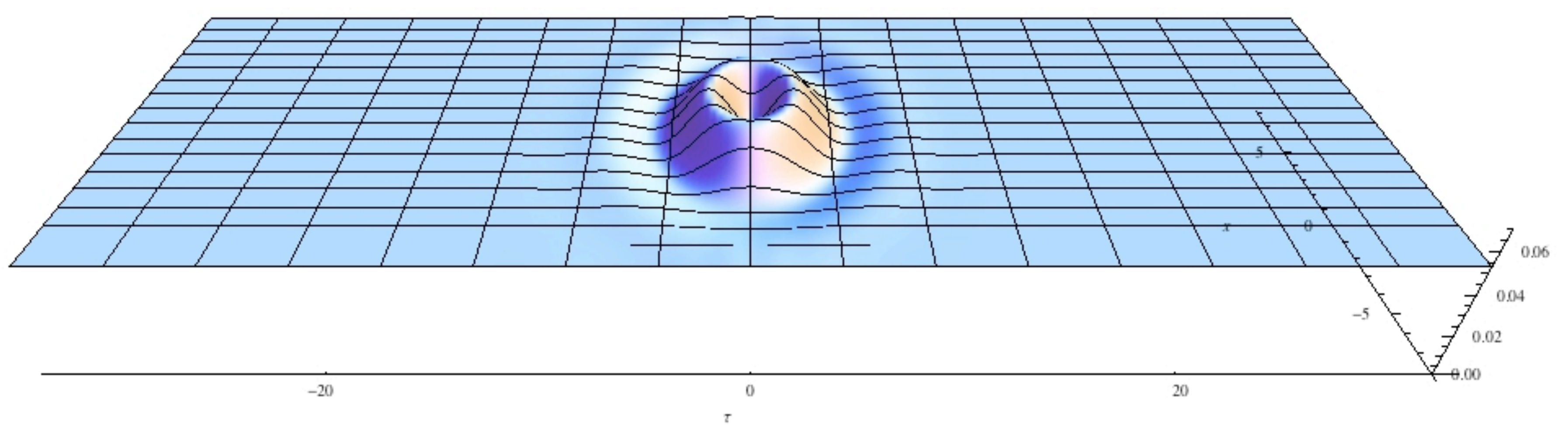
This is done by constructing the matrix corresponding to the modified electron operator.



# MPS for a localized quasi-particle

By summing over the delocalized quasi-particle states, using $P_{LLL}$, we obtain a localized quasi-particle!

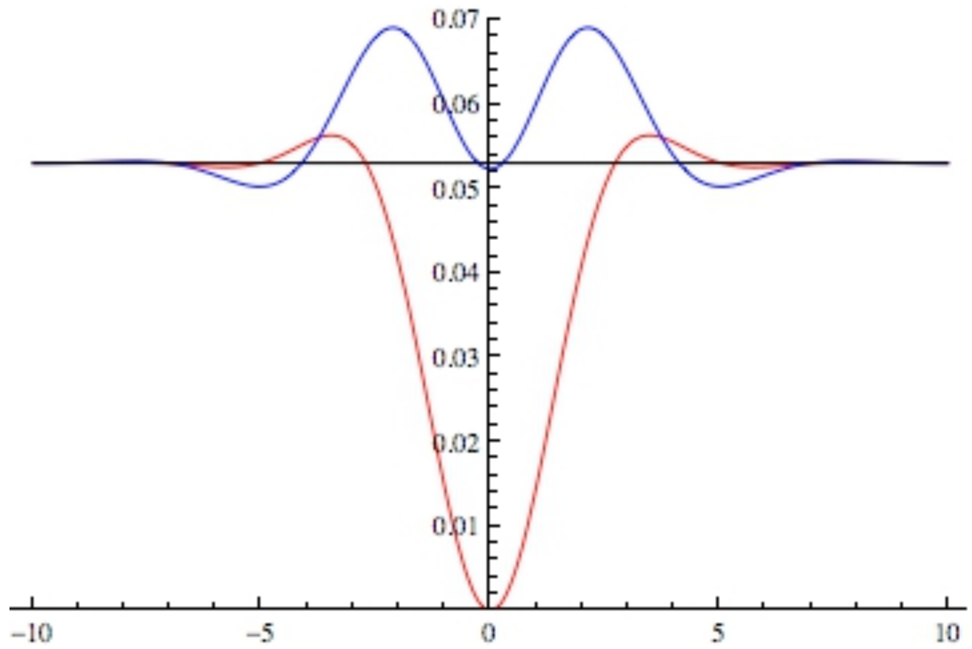


# MPS for a localized quasi-particle

The localized quasi-particle has cylinder symmetry, and it's charge integrates to -0.33344

Comparing the density of a quasi-hole and a quasi-particle:




Both are quite large, with a radius of about 7 $l_b$!

# Outlook

⋆ We successfully constructed quasi-particles states using MPS

⋆ Next steps:

- ✳ construct quasi-particle -- quasi-hole pair (not completely trivial!)
- ✳ Calculate braiding phases between particles and holes
- ✳ MPS for non-model quantum Hall states
- ✳ ...

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★ We successfully constructed quasi-particles states using MPS

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Thank you for your attention!