Conformal Loop Ensemble for the Integer Quantum Hall Transition

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Workshop on Geometric aspects of the Quantum Hall Effect
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Cologne, Germany

Work with I. Gruzberg.

Introduction

The Chalker-Coddington Network

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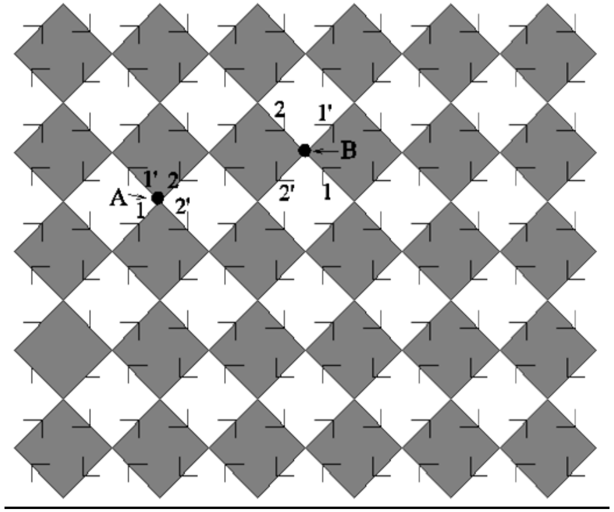

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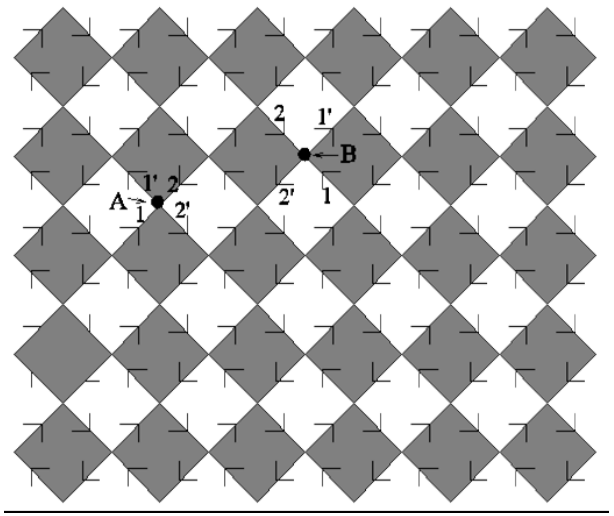

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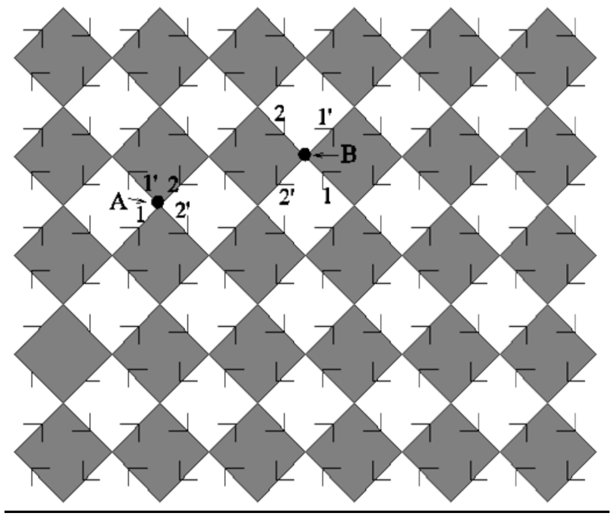

- The wavefunction accumulates a random phase on each link
- A beam splitter is at each node:

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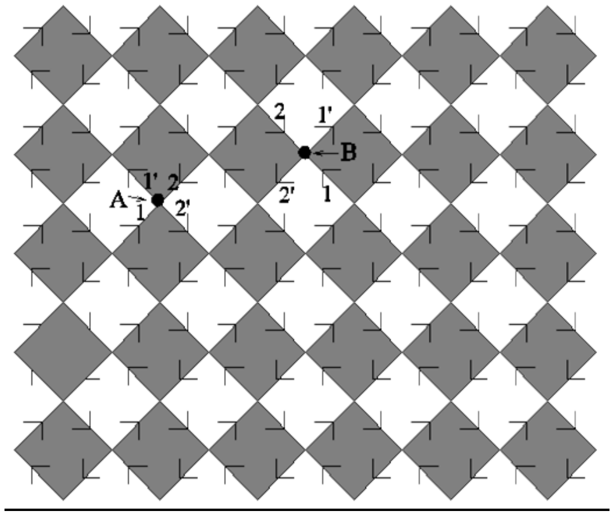

- The probability amplitude from a to b is:

$$G(a,b) = \sum_{P} \prod_{l \in P} e^{\imath\theta_l} \prod_{(l,l') \in P} S_{l,l'}$$

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Positive Weights

- We compute $\sigma(a,b) = \langle |G(a,b)|^2 \rangle = \sum_{P,P'} \langle W(P) W^*(P') \rangle$, with $W(P) = \prod_{l\in P} e^{\imath\theta_l} \prod_{(l,l')\in P} S_{l,l'}$.

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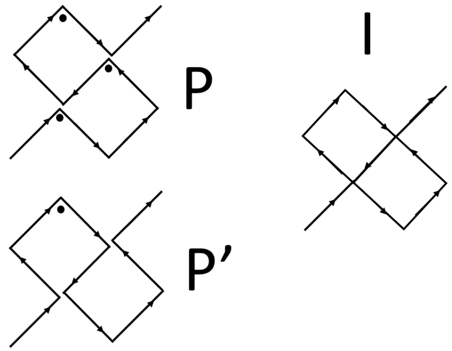

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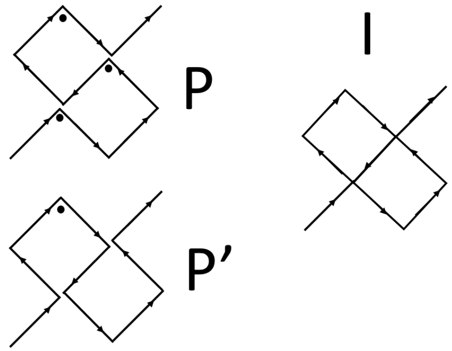
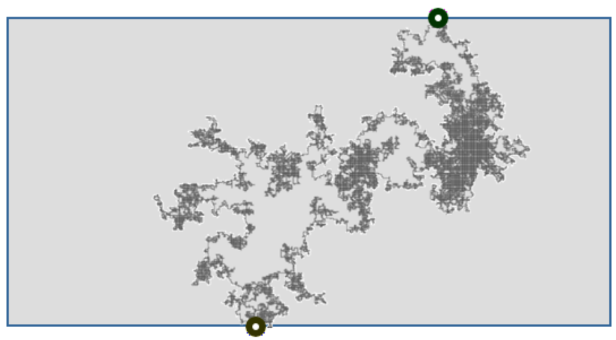


- We get $\left(S = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \right)$

$$\sigma(a,b) = \sum_I \frac{A(I)^2}{2^{L(I)}}, \quad A(I) = \sum_{P \in I} \prod_{(l,l') \in P} (-)^{\sigma(l,l')}$$

Conformal Restriction

- A Large Cluster's Shape is determined EB, I. Gruzberg, A. W. W. Ludwig by conformal invariance and intrinsic weights ('conformal restriction' Lawler)



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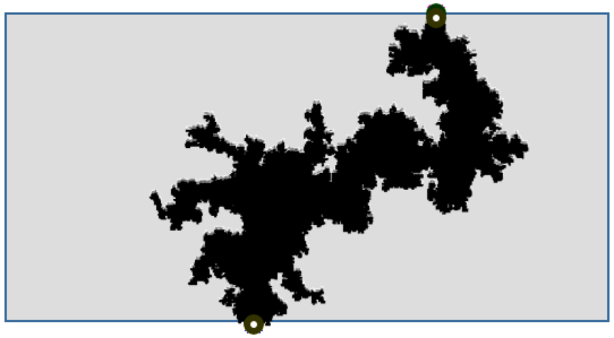

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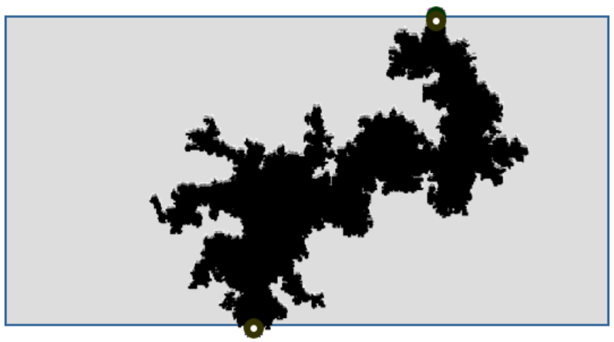

- Only external perimeter is determined.
- It's the same perimeter for Percolation, Random Walks, Transition, etc.

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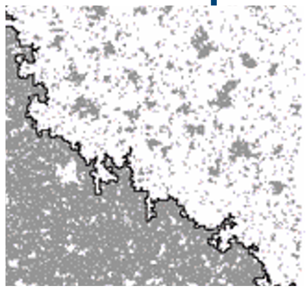
- The accessible fractal properties seem too general.
- A more detailed description of the interior of the path is needed.

Conformal Loop Ensemble Approach

Conformal Loop Ensembles

- In statistical mechanics, one can often describe the problem as that of loops in the plane.



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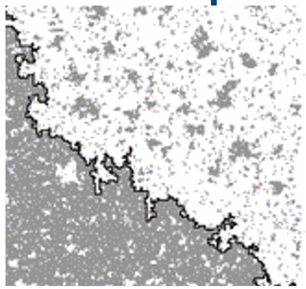

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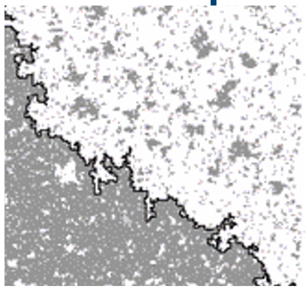

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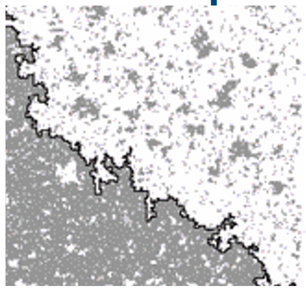

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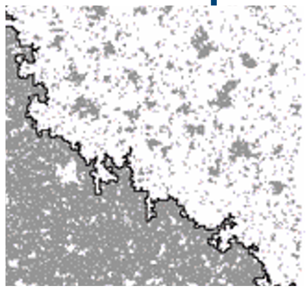

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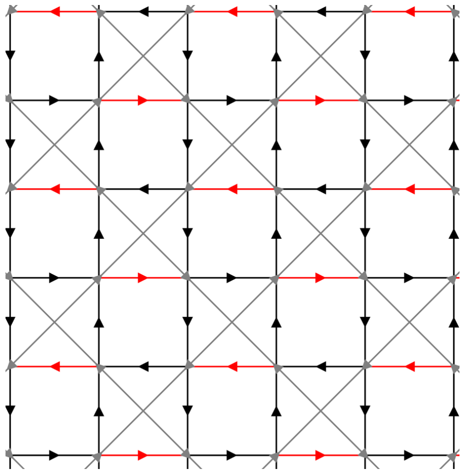
  1. Conformal Field Theory or
  2. Loop Ensembles and the Schramm-Loewner Equation (SLE).

The Manhattan (medial) Lattice

- It's easier to keep track of the sign on the median lattice, which in this case is the Manhattan lattice.



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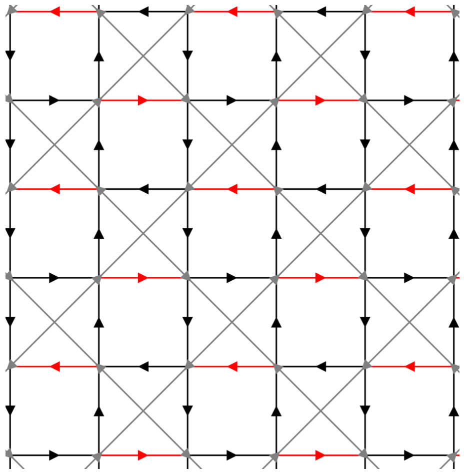

- We still need to count the ways to traverse the lattice, which interfere.

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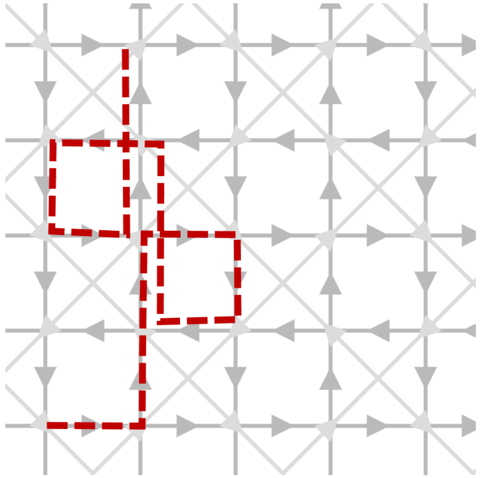

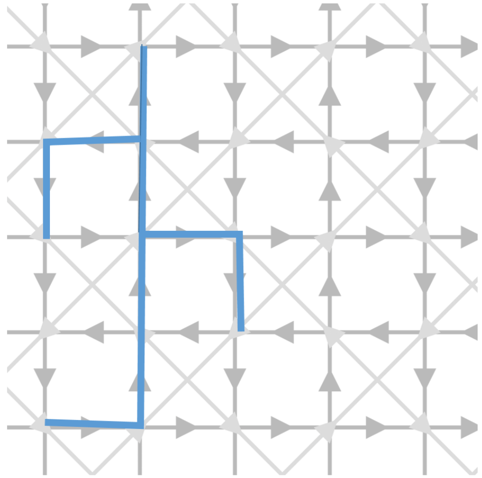

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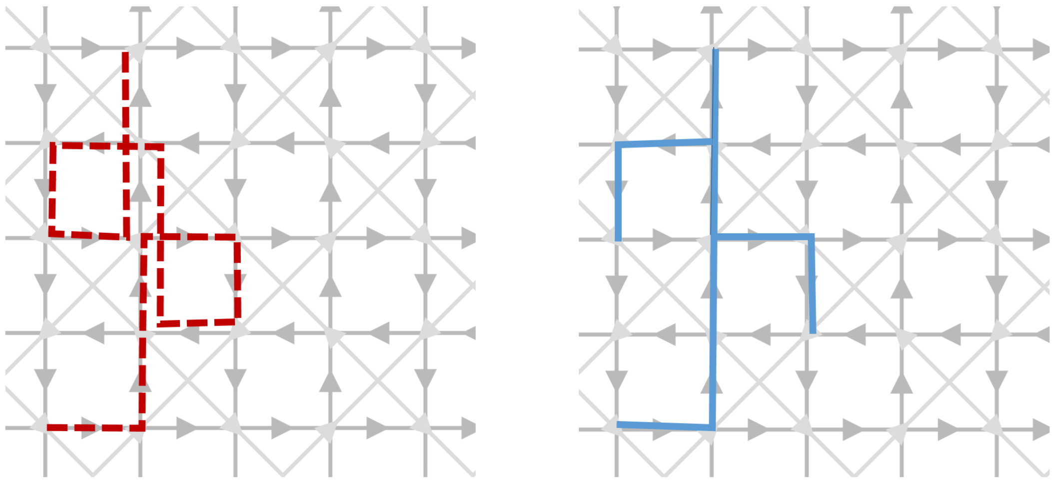

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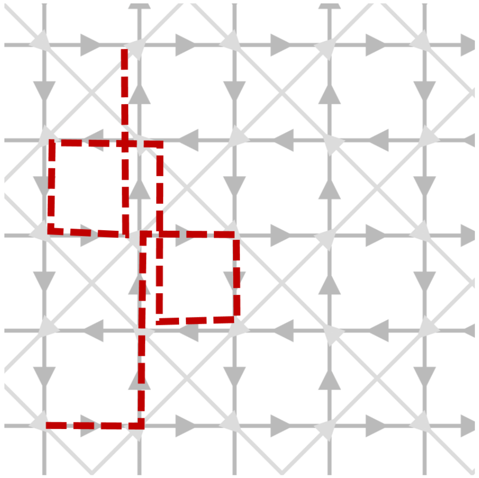

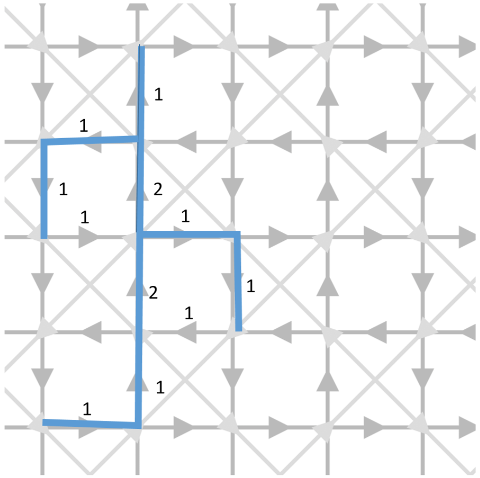

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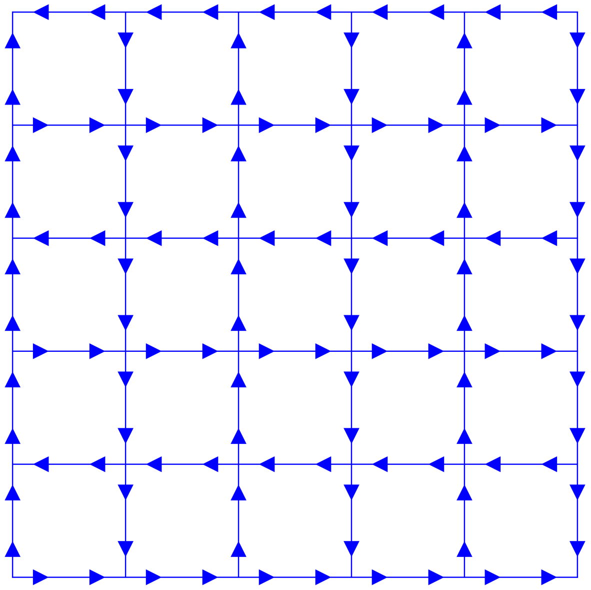

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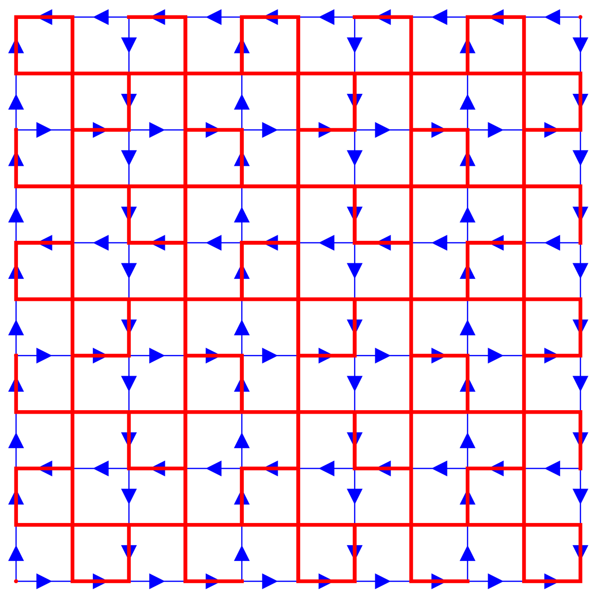

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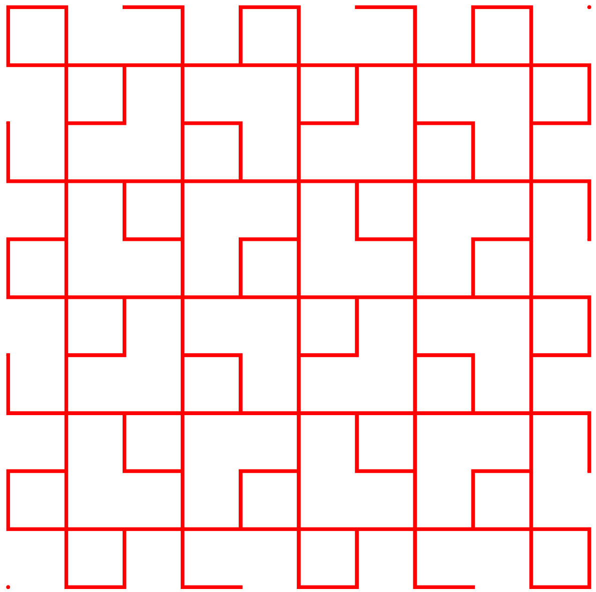

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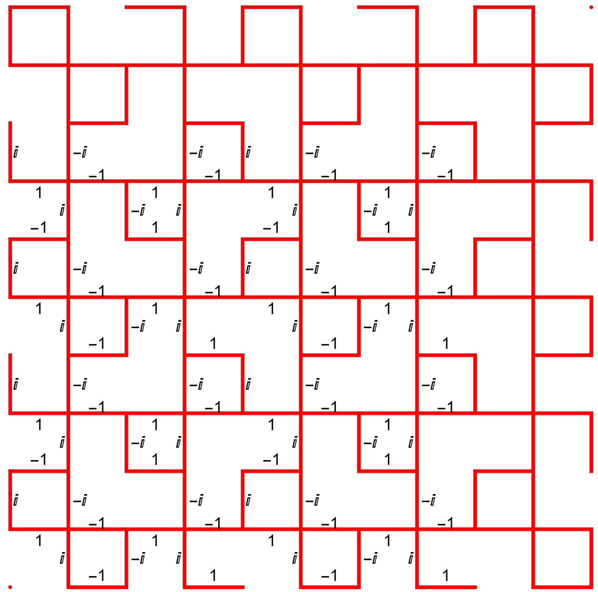

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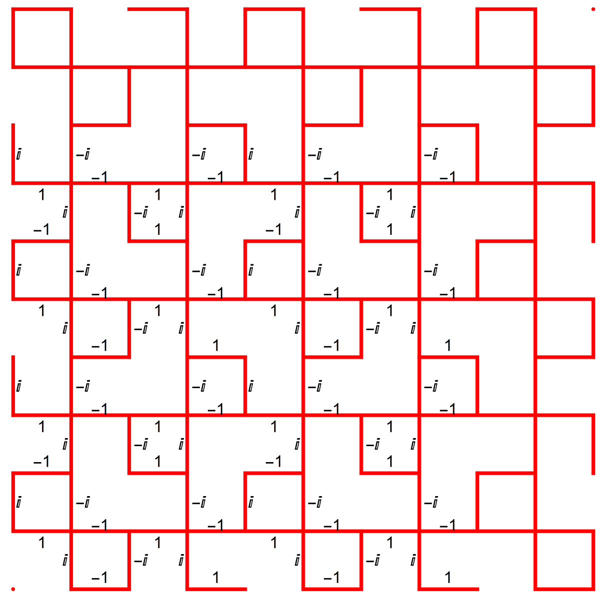
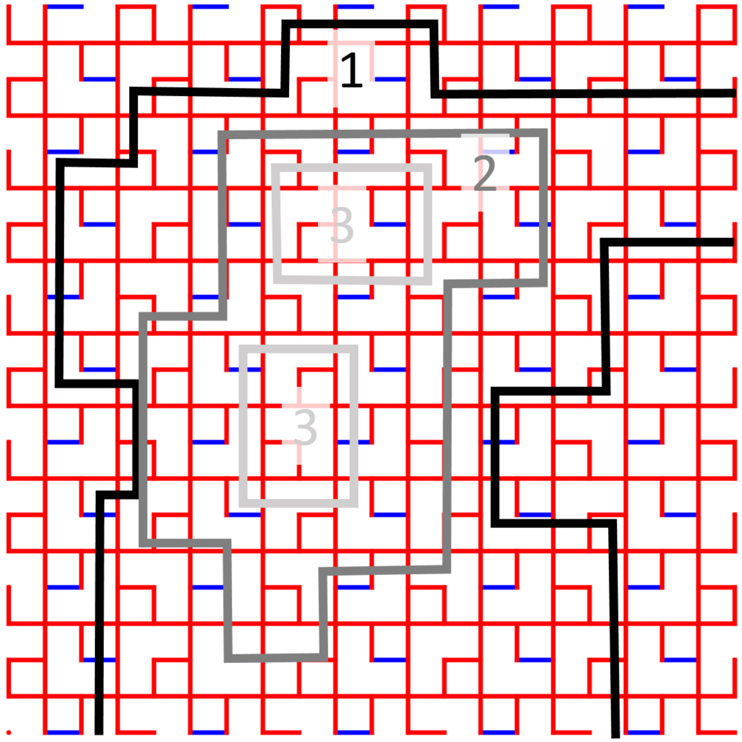


- $\langle \psi_{i_1}^{\dagger} \psi_{j_1} \psi_{i_2}^{\dagger} \psi_{j_2} \ldots \rangle = \det_{k,l} \left(K^{-1}(i_k, j_l) \right)$

The Loops

-



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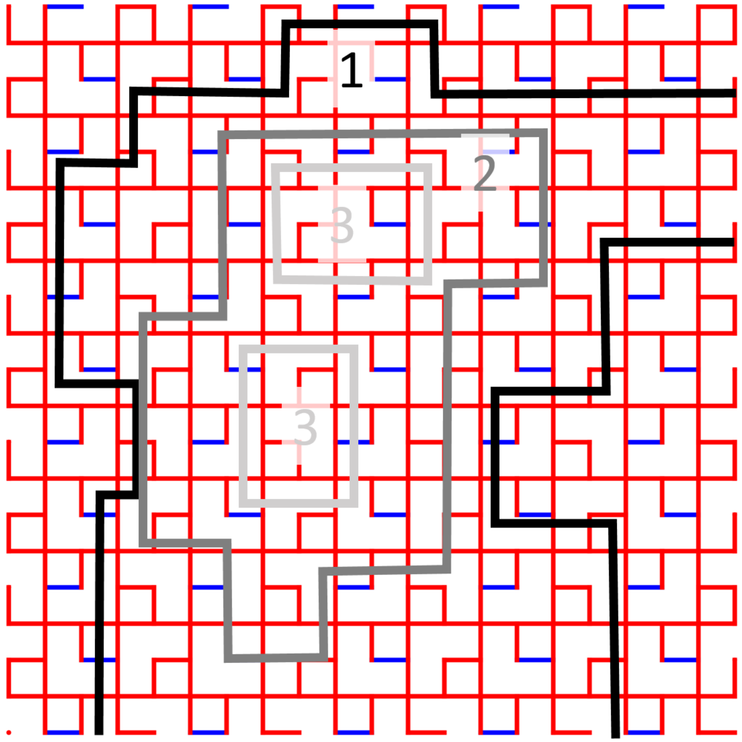
- 

- Each loop is weighted by two copies of signed restricted trees.

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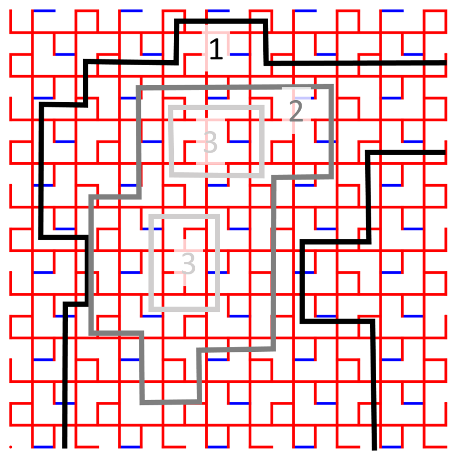

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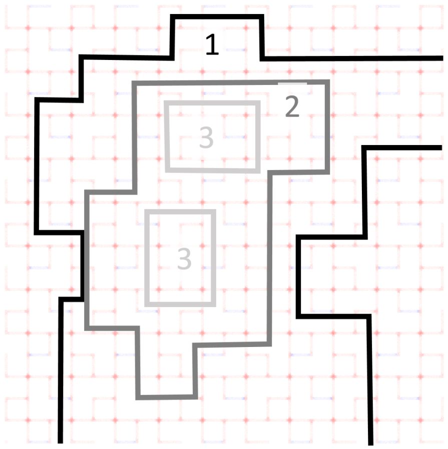

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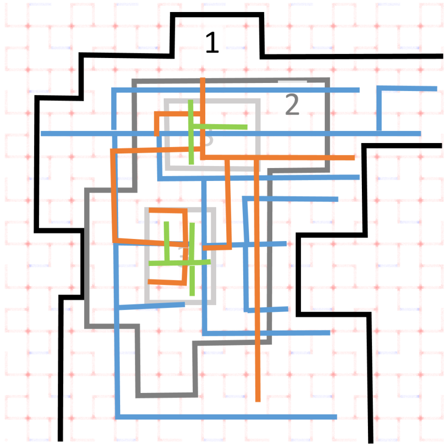

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- $Z = \det^2(\Delta)\langle e^{\varphi A \varphi}\delta(\partial\varphi - v)\rangle_{GFF} \stackrel{?}{=} \det^{2+\delta c}(\Delta)$

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- Critical exponents may be obtained as directly related to fractal dimensions of subsets of the loops and trees.