Effective field theory of the quantum Hall transition

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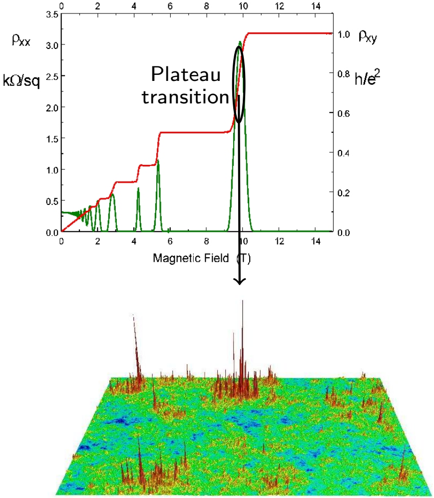
Geometric aspects of the QHE – 17/12/15

Joint work with D. Wieczorek & M. Zirnbauer (Köln)
Discussions with I. Gruzberg (OSU)

Refs: Phys. Rev. Lett. 112, 186803 (2014) + in preparation

Outline

Random geometry of quantum Hall transitions

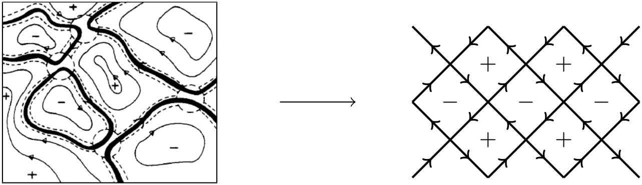


Integer quantum Hall effect:
2D electrons in magnetic field
and random electric potential

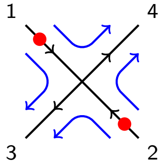
Question:
Universality of critical
wave functions (multifractal)

This talk: for first time we construct effective CFT

The Chalker–Coddington network model



- Wave function on the links
- Evolution operator $\mathcal{U}$

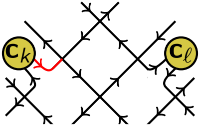


$$: \mathcal{U}_v = \begin{pmatrix} \sqrt{1-t^2} & t \\ -t & \sqrt{1-t^2} \end{pmatrix} \times \begin{pmatrix} e^{i\phi_1} & 0 \\ 0 & e^{i\phi_2} \end{pmatrix}$$

- ϕ's random phases. Disorder average $\mathbb{E}(\cdot)$
- $t_c^2 = 1/2$: plateau transition

Point contacts and the operator K

- Open network



- Hilbert space $= \mathcal{H}_{\text{contact}} \oplus \mathcal{H}_{\text{bulk}}$
- Projectors:
  $P = \sum_{k=1}^{n} |\mathbf{c}_k\rangle\langle\mathbf{c}_k|$, $Q = 1 - P$
- $\mathcal{U} \to Q\mathcal{U}$, prob. loss

- Key object:

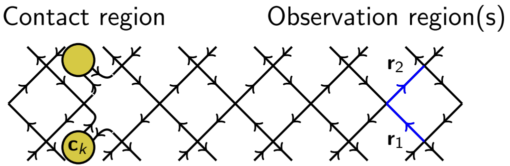
$$K = Q\mathcal{U}(\mathbf{1} - Q\mathcal{U})^{-1} + (\mathbf{1} - \mathcal{U}^{-1}Q)^{-1}$$

Rem 1: $\operatorname{rank}(K) =$ number of contacts:

$$K = (\mathbf{1} - Q\mathcal{U})^{-1}\left[Q\mathcal{U}(\cancel{\mathbf{1}} - \mathcal{U}^{-1}Q) + (\mathbf{1} - \cancel{Q\mathcal{U}})\right](\mathbf{1} - \mathcal{U}^{-1}Q)^{-1}$$
$$= (\mathbf{1} - Q\mathcal{U})^{-1} P (\mathbf{1} - \mathcal{U}^{-1}Q)^{-1}$$

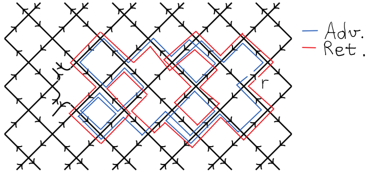
Rem 2: $\mathbb{E}(K) = \mathbf{1}$

The observables



$\rightarrow$ Observables: *statistics of eigenvalues of* $K\big|_{Obs}$

- Example: 1 Obs. link: $\mathbb{E}\left(\langle \mathbf{r}|K|\mathbf{r}\rangle^q\right)$
  - $\mathbb{E}\left(\langle \mathbf{r}|K|\mathbf{r}\rangle\right) = 1$
  - $\mathbb{E}\left(\langle \mathbf{r}|K|\mathbf{r}\rangle^2\right) = 1 + 2\mathbb{E}\left(|\langle \mathbf{r}|Q\mathcal{U}(1 - Q\mathcal{U})^{-1}|\mathbf{r}\rangle|^2\right)$



  - One can prove symmetry $\mathbb{E}\left(\langle \mathbf{r}|K|\mathbf{r}\rangle^q\right) = \mathbb{E}\left(\langle \mathbf{r}|K|\mathbf{r}\rangle^{1-q}\right)$

Interpretation in terms of scattering states

- Time evolution open network

$$|\psi(t+1)\rangle = Q\mathcal{U}\left(|\psi(t)\rangle + |\psi_{\text{in}}\rangle\right)$$

in-wave bdry condition

- Scattering states are the stationary states:

$$|\psi_{\mathbf{c}}\rangle = Q\mathcal{U}(1 - Q\mathcal{U})^{-1}|\mathbf{c}\rangle\,.$$

⇒ Recalling $K = (\mathbf{1} - Q\mathcal{U})^{-1}P(\mathbf{1} - \mathcal{U}^{-1}Q)^{-1}$, in the bulk:

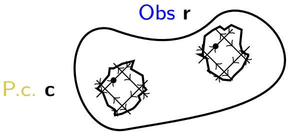
$$K = \sum_{i=1}^{n} |\psi_{\mathbf{c}}\rangle\langle\psi_{\mathbf{c}}|$$

In particular if 1 pt contact

$$\langle\mathbf{r}|K|\mathbf{r}\rangle = |\psi_{\mathbf{c}}(\mathbf{r})|^2$$

Result I: Pure scaling

2 pt functions

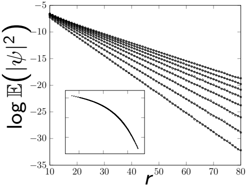


After coarse graining:

$$\mathbb{E}\left\{|\psi_{\mathbf{c}}(\mathbf{r})|^{2q}\right\} = \left\langle e^{q\varphi(\mathbf{r})}\pi(\mathbf{c})\right\rangle_{\mathrm{CFT}}$$

$$\overset{\text{Plane}}{\propto} |\mathbf{r}-\mathbf{c}|^{-2\Delta_q}$$

- $e^{q\varphi}$ spinless primary, dim: $\Delta_q = \Delta_{1-q}$
- π *not* pure scaling, by orthogonality contributes Δ_{1-q}
  $\rightarrow$ "background charge" $= 1$.
- Numerical check cylinder: ✓



Result II: Abelian fusion

3 pt functions

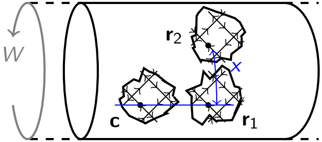
$$\mathbb{E}\left\{|\psi_{\mathbf{c}}(\mathbf{r}_1)|^{2q_1}|\psi_{\mathbf{c}}(\mathbf{r}_2)|^{2q_2}\right\} = \left\langle \pi(\mathbf{c}) e^{q_1\varphi(\mathbf{r}_1)} e^{q_2\varphi(\mathbf{r}_2)} \right\rangle_{\mathsf{CFT}}$$

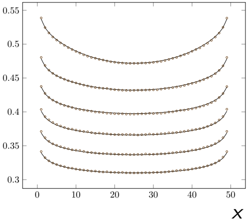
- Claim: Abelian (Virasoro) fusion:

$$e^{q_1\varphi} \times e^{q_2\varphi} = e^{(q_1+q_2)\varphi}$$

$\Rightarrow$ π contributes $\Delta_{1-(q_1+q_2)}$: 3pt function fixed by conformal inv.

- Numerics: ✓





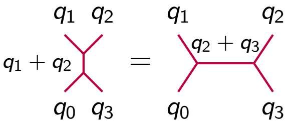
Corollary: Parabolicity

4 pt functions

$$\mathbb{E}\left\{|\psi_{\mathbf{c}}(\mathbf{r}_1)|^{2q_1}|\psi_{\mathbf{c}}(\mathbf{r}_2)|^{2q_2}|\psi_{\mathbf{c}}(\mathbf{r}_3)|^{2q_3}\right\} = \left\langle \pi(\mathbf{c}) e^{q_1\varphi(\mathbf{r}_1)} e^{q_2\varphi(\mathbf{r}_2)} e^{q_3\varphi(\mathbf{r}_3)} \right\rangle_{\mathrm{CFT}}$$

$\uparrow$ Δ_{q_0}, $q_0 \equiv 1 - (q_1 + q_2 + q_3)$

On the plane, crossing symmetry (one conformal block)



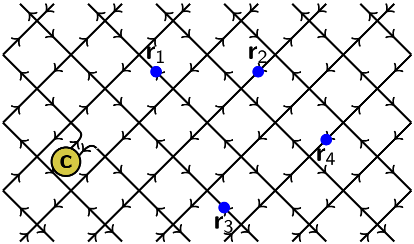
$\Rightarrow$ $\boxed{\Delta_q = Xq(1-q)}$

with X undetermined. Numerically, $X \sim \frac{1}{4}$.

Rem For a closed system (LDOS), see also [Suslov '15] vs [Evers *et al.* '08, Obuse *et al.* '08]

Central result: effective theory

Crossing symmetry fixes the 4 pt function and the effective CFT is a free boson with background charge 1.

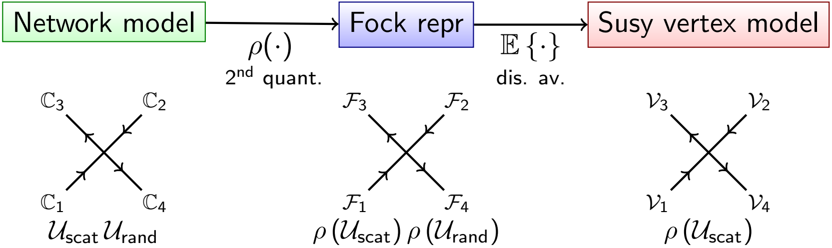


⇒ Prediction for the plane:

$$\mathbb{E}\left(|\psi_{\mathbf{c}}(\mathbf{r}_1)|^{2q_1}\cdots|\psi_{\mathbf{c}}(\mathbf{r}_N)|^{2q_N}\right)=\left\langle \pi(\mathbf{c})e^{q_1\varphi(\mathbf{r}_1)}\cdots e^{q_N\varphi(\mathbf{r}_N)}\right\rangle_{\mathrm{CFT}}$$
$$=\prod_j|\mathbf{c}-\mathbf{r}_j|^{-2Xq_j(1-\sum_i q_i)}\prod_{i<j}|\mathbf{r}_i-\mathbf{r}_j|^{-2Xq_iq_j}\,.$$

From network to vertex models I

Second quantization



- $\pm$ bosons & fermions:

$$\rho(e^X) = \exp\begin{pmatrix} b_+^\dagger & f_+^\dagger & -b_- & f_- \end{pmatrix} \mathbf{1}_4 \otimes X \begin{pmatrix} b_+ \\ f_+ \\ b_-^\dagger \\ f_- \end{pmatrix}$$

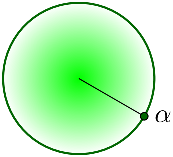
- Point contacts: $\rho(Q) = \pi(\mathbf{c}) \equiv (|0\rangle\langle 0|)(\mathbf{c})$

From network to vertex model II

Mapping of observables

- Def. $B = b_+ - e^{i\alpha} b_-^\dagger$

Boundary of Bogoliubov transf.



$\Rightarrow$

1. $[B, B^\dagger] = 0$
2. $B^\dagger B > 0$

$$\Rightarrow \quad |\psi_{\mathbf{c}}(\mathbf{r})|^{2q} \overset{\text{Wick}}{=} q!^{-1} \left\langle (B^\dagger B)^q(\mathbf{r})\, \pi_0(\mathbf{c}) \right\rangle_{\mathcal{F}}$$

- $e^{q\varphi}$ are highest weight vectors of symmetry superalgebra
  - $\Rightarrow$ Scaling field
  - $\Rightarrow$ Abelian OPE

From network to vertex model II

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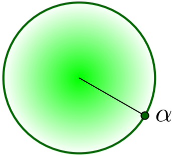

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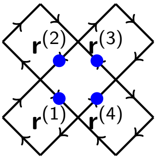
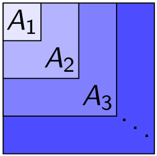
The compact sector

Recall $K = \sum_k |\psi_{\mathbf{c}_k}\rangle\langle\psi_{\mathbf{c}_k}|$. Consider princ. minors at Obs region;

Obs



$K\big|_{\text{Obs}} =$ 

- $A_n \equiv \text{Det}\left(K\big|_{\text{Obs}}\right)_{n\times n} \xrightarrow[\mathbf{c}_k \to \mathbf{c}]{\mathbf{r}^{(i)} \to \mathbf{r}} \langle V_n(\mathbf{r})\pi(c)\rangle_{\text{CFT}}$
- $V_n(\mathbf{r})$ primary, dim $= \tilde{\Delta}_n$, $\tilde{\Delta}_1 = 0$
- In VM, fermionic highest weight vector:

$$V_n = e^{in\vartheta}, \quad \tilde{\Delta}_n = Xn(n-1)$$

Numerical checks + [Gruzberg,Mirlin,Zirnbauer '13]

Question: "parent" theory $S[\varphi, \vartheta, \dots]$?

Conclusions

- New microscopic approach to CFT of quantum Hall transition
- Derived first time theory of critical wave functions: free boson
- Methods general and allow classification of critical behaviors at Anderson transitions in 2D.
  Eg SQHE [Gruzberg,Read,Ludwig '99], $\Delta_q = \frac{1}{4}q(3-q)$
- Outlook:
  - Theory on torus
  - Conserved currents on the lattice/CFT

Thank you!

Conclusions

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