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Goldstone mode stochastization in a quantum Hall ferromagnet

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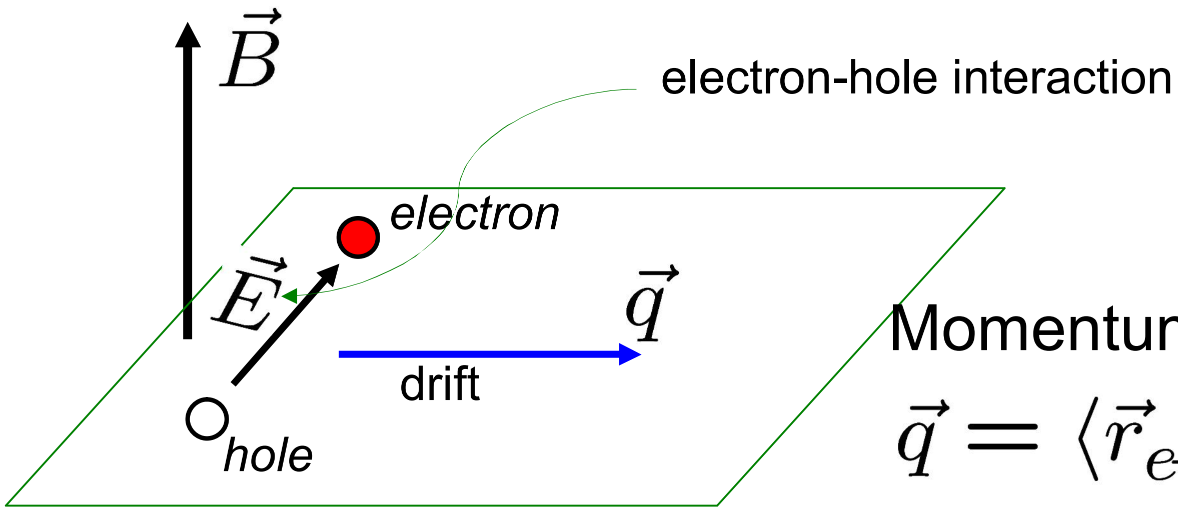
(Cologne, December 14, 2015)

OUTLINE

- “Zero” and “nonzero” spin excitons are lowest energy excitations in a spin polarized integer quantum Hall system
- Goldstone mode excitation -- *experiment*
- Goldstone mode -- *theory*
- Goldstone mode stochastisation – *theory and experiment*

Magneto-exciton (Gor'kov, Dzyaloshinskii, 1967)



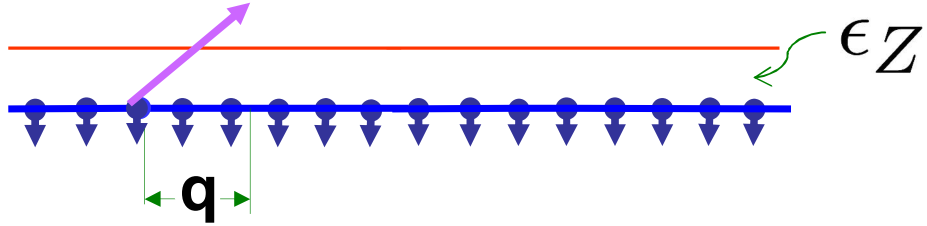
Momentum:

$$\vec{q} = \langle \vec{r}_{e\text{–}h} \rangle \times \hat{z} / l_B^2$$

- The Landau-level degeneration is lifted due to electron-hole interaction. 'Natural' quantum number $\vec{q}$ appears due to the translation invariance of the system.
- Due to translation invariance a special $\hat{P}$ operator appears playing the same role as momentum operator in the absence of magnetic field: $[\hat{P}, H_{\text{int}}] \equiv 0$.
- The magneto-exciton energy: $E_X = \mathcal{E}(q)$

Simplest excitation in a completely polarized QHF

Spin exciton (spin wave): $\delta n = 0, \quad \delta S_z = -1$



First-order approximation results:
(Yu. A Bychkov *et al*, 1981;
C. Kallin, B.I. Halperin 1984)

$$E_{\rm sw}(q) = \epsilon_{\rm Z} + \mathcal{E}_{\rm sw}(q)$$

Spin exciton and other exciton-type states are suitable to study by using the excitonic basis set: $\mathcal{Q}^{\dagger}_{ab\mathbf{q}}|0\rangle$ **,** $\quad |0\rangle \equiv |{\rm vac}\rangle$

$$\mathcal{Q}^{\dagger}_{ab\mathbf{q}} = N_{\phi}^{-1/2} \sum_{p} e^{-ipq_x}\, b^{\dagger}_{p+q_y/2} a_{p-q_y/2}\,,$$

a and b are binar indexes: $a = (n_a, \sigma_a)$

the LL index

spin index: $\sigma = \uparrow / \downarrow$

Important: Exciton states are eigen states of the Gor'kov-Dzyaloshinski *P*-momentum operator of the magnetoexciton:

$$\hat{P}\mathcal{Q}^{\dagger}_{ab\mathbf{q}}|0\rangle = \mathbf{q}\mathcal{Q}^{\dagger}_{ab\mathbf{q}}|0\rangle$$

Zero spin exciton if $\mathbf{q} \equiv 0$:

$$\mathcal{Q}_{\mathbf{0}}^{\dagger} \equiv S_x - iS_y$$

$\mathcal{Q}_{\mathbf{0}}^{\dagger}|0\rangle$ **is eigen state for any QH system corresponding to the change of spin numbers**

$\delta S_z = -1$ **and** $\delta S = 0$

Nonzero spin exciton

$$[H_{\text{int}}, \mathcal{Q}_{\mathbf{q}}^{\dagger}]|0\rangle = \mathcal{E}_{\text{sw}}(q)\mathcal{Q}_{\mathbf{q}}^{\dagger}|0\rangle$$

is eigen state for the QH ferromagnet to the leading order approximation in H_{int} **.**
Change of the spin numbers is

$$\delta S_z = \delta S = -1$$

Important: **in spite of** $\mathcal{Q}_{\mathbf{0}}^{\dagger} \equiv \lim_{\mathbf{q}\to 0}\mathcal{Q}_{\mathbf{q}}^{\dagger}$

these are different states:

$$\mathcal{Q}_{\mathbf{0}}^{\dagger}|0\rangle \neq \lim_{\mathbf{q}\to 0}\mathcal{Q}_{\mathbf{q}}^{\dagger}|0\rangle$$

Physical meaning of this “discrepancy”

Let $\mathcal{L}$ **be linear dimension of the system.**

$2\pi/\mathcal{L}$ **is spacing for** q **numbers**

$\mathbf{q} \equiv 0$ $\longrightarrow$ **means that** $q << 2\pi/\mathcal{L}$

whereas

$q \to 0$ $\longrightarrow$ **means that** $2\pi/\mathcal{L} << q << 1/l_B$

Spin Goldstone mode

Ground state:

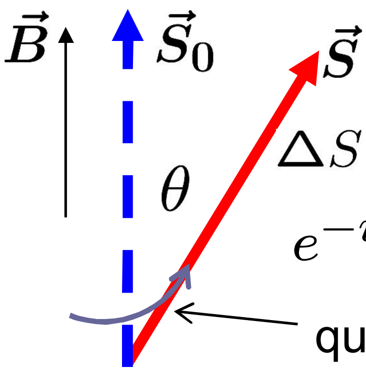
$$|0,0\rangle = |\overbrace{\uparrow\uparrow\uparrow\ldots\uparrow}^{\mathcal{N}_\phi}\rangle$$ ($\mathcal{N}_\phi$ is the degeneration number of the completely occupied Landau level).

The eigenstate

$|N,0\rangle = (\mathcal{Q}_0^\dagger)^N|0,0\rangle$, where $\mathcal{Q}_0^\dagger = S_-/\sqrt{\mathcal{N}_\phi}$ ($S_- = S_x - iS_y$)

is the Goldstone condensate: $S = \mathcal{N}_\phi/2$ and $S_z = \mathcal{N}_\phi/2 - N$

$$\cos\theta = S_z/S$$

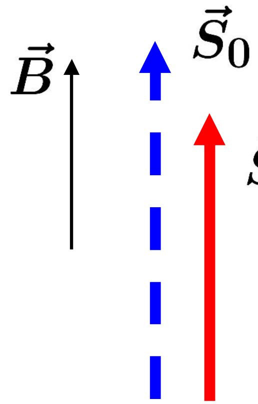


$\Delta S = 0 \quad \Delta S_z \neq 0$

$e^{-iE_N t}|N,0\rangle E_N = \epsilon_Z N$

quantum precession

Another case of deviation is ensemble of nonzero spin excitons:



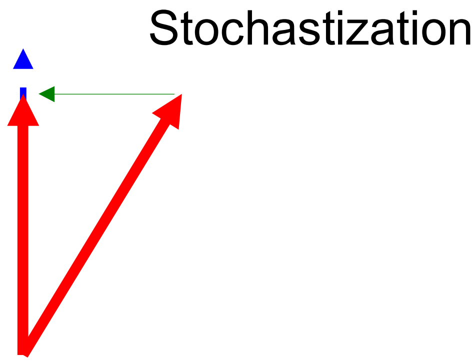
$\Delta S = \Delta S_z$

Relaxation of spin excitations

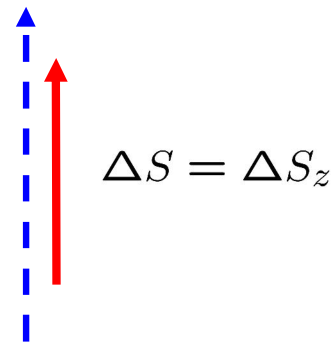
Spin stochastization or spin relaxation?

Total number of spin excitons determines the S_z spin component: $S_z = N_\phi/2 - N$

Spin stochastization of the GM is a fast process without change of the S_z spin component , i.e. at fixed number of spin excitons N and at fixed Zeeman energy.

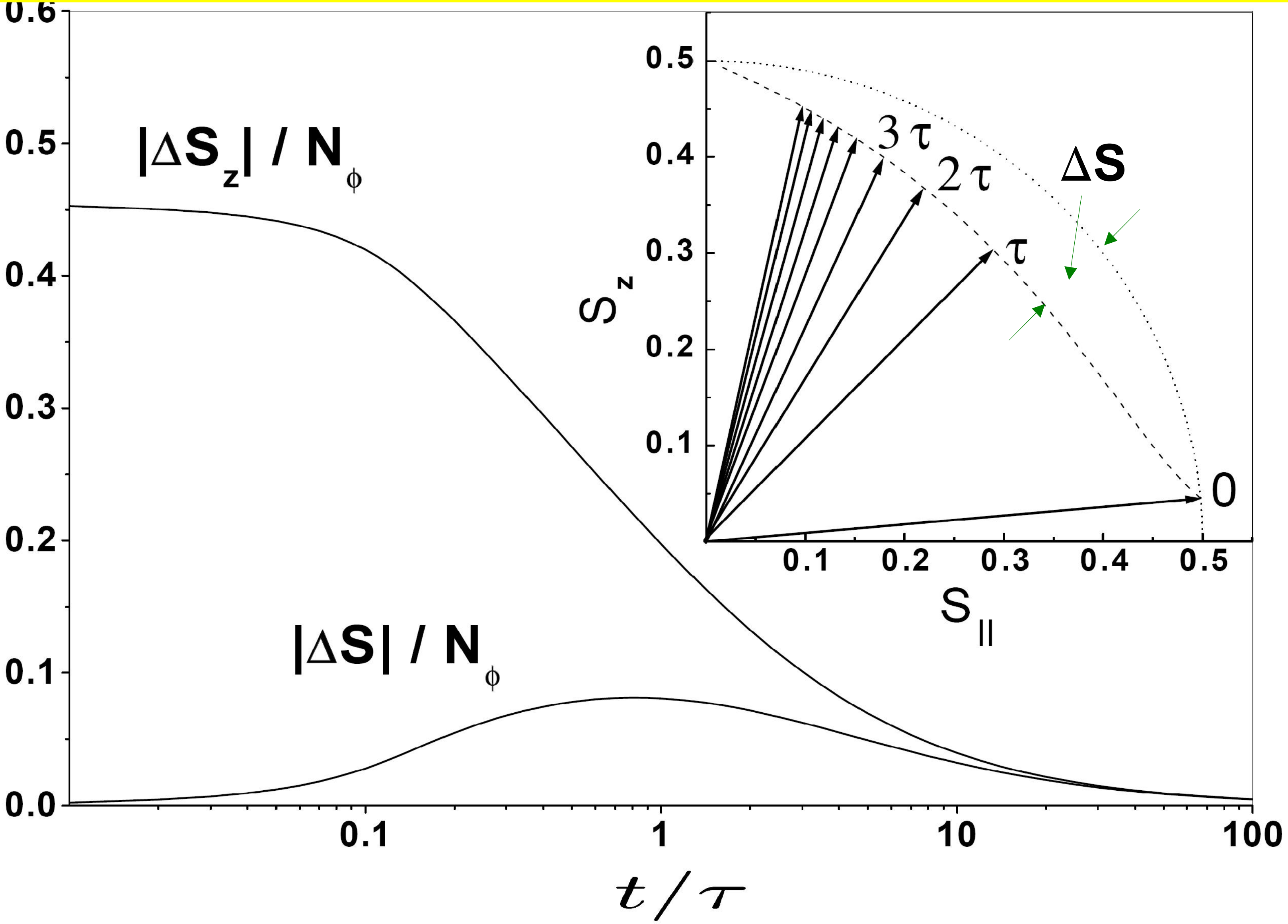


Spin exciton relaxation is change of the S_z component --- occurs much slower because related to annihilation processes of spin excitons and release of the energy --- studied experimetally and theoretically (Zhuravlev,SD, Kulik, Kukushkin, PRB 2014). The time > 100 ns .



The GM decay if spin relaxation and stochastization occur simultaneously.

If a single mechanism for relaxation and stochastization -> SO coupling + smooth random potential (SD PRL 2004)



An elementary stochastisation process:

$$|N,0\rangle \Rightarrow |N,\mathbf{q}\rangle = (\mathcal{Q}_0^\dagger)^{N-1}\mathcal{Q}_\mathbf{q}^\dagger|0,0\rangle,$$

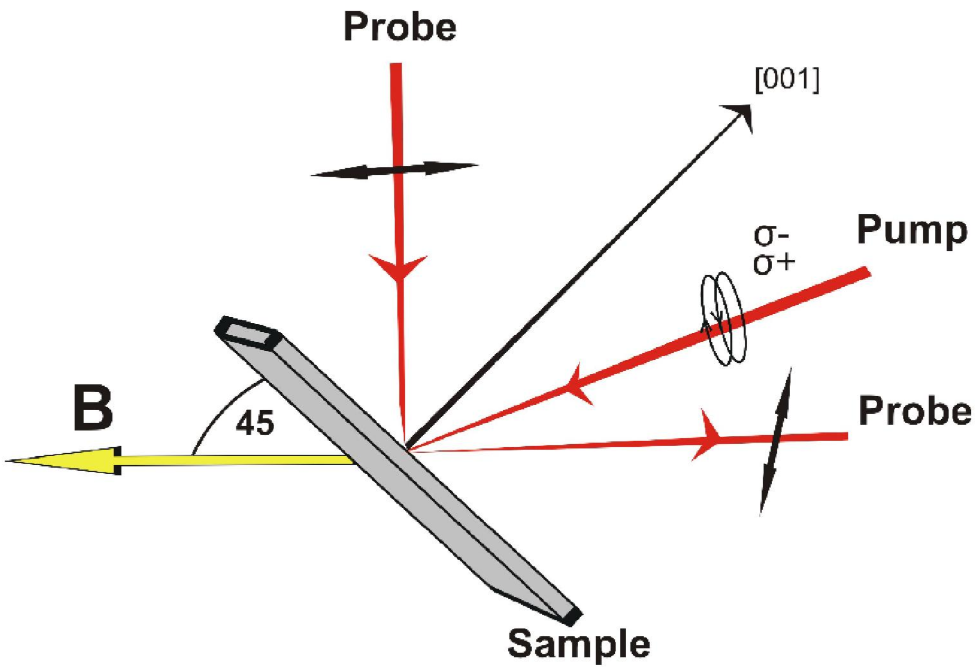
where $\mathcal{Q}_\mathbf{q}^\dagger = \mathcal{N}_\phi^{-1/2}\sum_p e^{-iq_xp}b^\dagger_{p+\frac{q_y}{2}}a_{p-\frac{q_y}{2}}$, $a{=}\uparrow$ and $b{=}\downarrow$.

Both $|N,0\rangle$ and $|N,\mathbf{q}\rangle$ are *eigenstates*. orthogonal due to the translation invariance.

The $|N,0\rangle \to |N,\mathbf{q}\rangle$ transition occurs without a change in the $S_z{=}\mathcal{N}_\phi/2-N$ component.

At $q \to 0$, the energies of both states (E_N) are the same. However, the states $|N,0\rangle$ and $|N,\mathbf{q}\rangle$ remain different even at $q \to 0$ and have different: $S=\mathcal{N}_\phi/2$ and $S=\mathcal{N}_\phi/2-1$, respectively.

Experiment: the time-resolved Kerr technique is used



The transverse component perpendicular to magnetic field

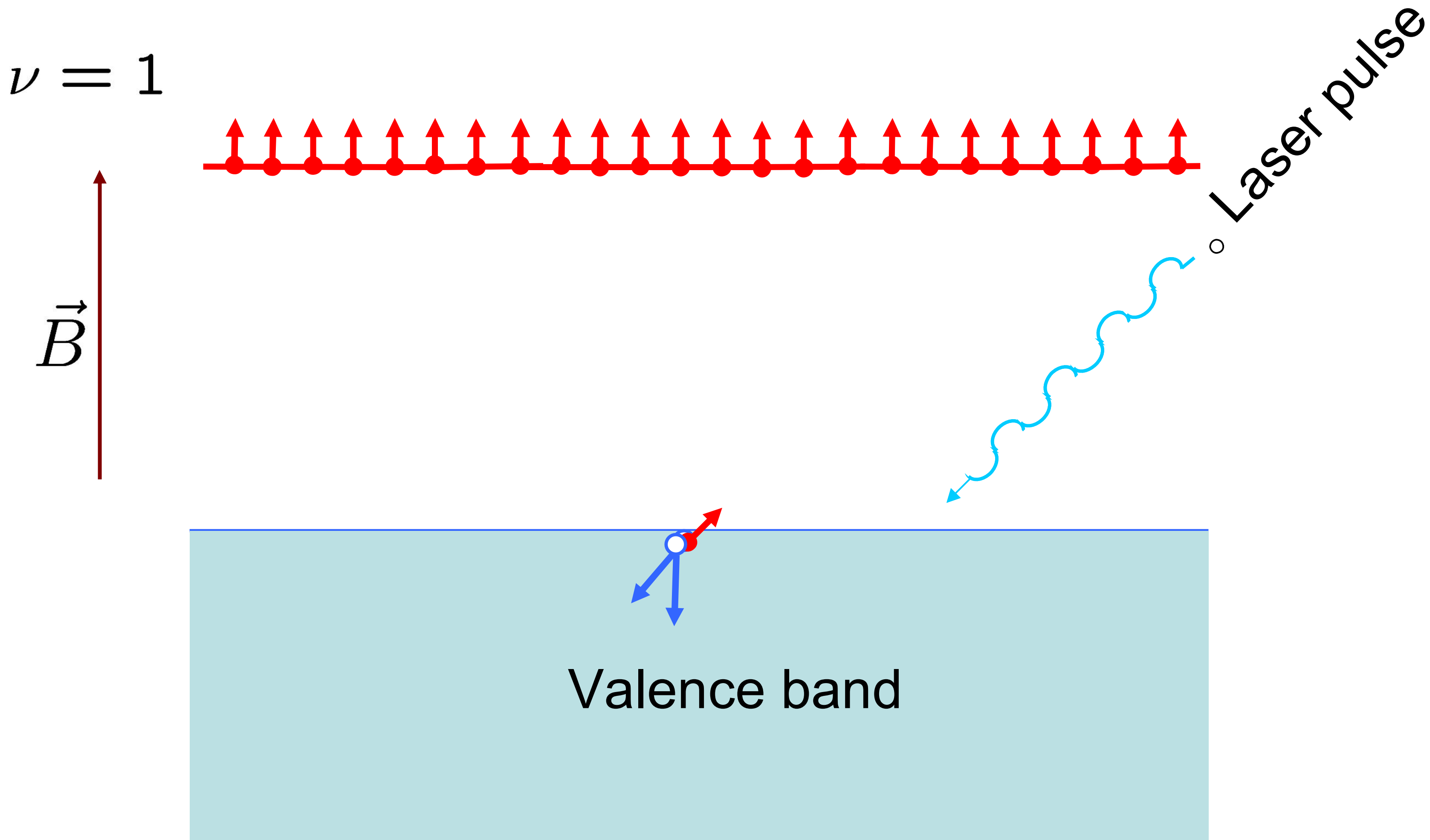
$$\langle S_x(t) + iS_y(t) \rangle$$

is measured

Quantum mechanical averaging

A. Larionov, L. Kulik, SD, I Kukushkin. PRB 2015

How is the Goldstone mode is excited experimetally (elementary transition process)



The present experimental situation

If a single electron, then instead of $\uparrow = \binom{1}{0}$ we have $\nearrow = \begin{pmatrix} \cos\frac{\beta}{2} \\ -\sin\frac{\beta}{2} \end{pmatrix}$

If one photon is absorbed in the state $|0,0\rangle = |\uparrow\uparrow\uparrow \ldots \uparrow\rangle$, then initially we have a combination of vectors

$$|\nearrow\uparrow\uparrow\uparrow\ldots\uparrow\rangle,\ |\uparrow\nearrow\uparrow\uparrow\ldots\uparrow\rangle,\ |\uparrow\uparrow\nearrow\uparrow\ldots\uparrow\rangle, \ldots \text{ and } |\uparrow\uparrow\uparrow\ldots\uparrow\nearrow\rangle,$$

On the base of the indistinguishability principle one finds the initial state

$$|\mathrm{i}\rangle = \hat{L}_\beta(0)|0,0\rangle, \text{ where } \hat{L}_\beta(0) = \cos\frac{\beta}{2}\hat{I} - \sin\frac{\beta}{2}\mathcal{Q}_0^\dagger$$

The appearance of a zero-exciton operator is stipulated by the strict \`verticality' of the transition process

$$\mathcal{L}k_{\mathrm{photon}\|} \ll 1,$$

where $\mathcal{L}$ is a linear characteristic of 2D density spatial fluctuations

Why only one `tilted' electron?

The initial state is not an eigenstate. It does not correspond to definite S_z but still corresponds to definite $S = \mathcal{N}_\phi/2$.

Under the experimental conditions, $N \ll N_\phi$, the elementary dephasing process is a single exciton process.

Consider a domain of area A smaller than $\mathcal{A}_{\text{sp}}/N$: $A \ll \mathcal{A}_{\text{sp}}/N$ ($\mathcal{A}_{\text{sp}}$ is the area of the laser spot).

No more than a single photon is absorbed within the A domain,

Kerr oscillations

Our task is to study the temporal evolution of the initial state

$$|\mathrm{i}\rangle = \cos\frac{\beta}{2}|0,0\rangle - \sin\frac{\beta}{2}|1,0\rangle$$

.

In the absence of any violation of the translation invariance, the Schroedinger equation results in state

$$|t\rangle = \hat{L}_\beta(t)|0,0\rangle$$ at moment t, where

$$\hat{L}_\beta(t) = \cos\frac{\beta}{2}\hat{I} - \sin\frac{\beta}{2}e^{-i\epsilon_Z t}\mathcal{Q}_0^\dagger$$

The calculation of expectation $\langle t|\hat{S}_x + i\hat{S}_y|t\rangle = -\frac{1}{2}\sin\beta\sqrt{\mathcal{N}_\phi}e^{-i\epsilon_Z t}$

explains the Kerr signal oscillations with frequency $\epsilon_Z/\hbar$

but does not explain the Kerr signal decay.

The stochastization process (slow compared to the precession)

This is a conversion of component $e^{-i\epsilon_Z t}|1,0\rangle$ of state $|t\rangle$

to component $e^{-i\epsilon_Z t}|1,\mathbf{q}\rangle$ at $q \to 0$.

When calculating the $\langle t,\mathbf{q}|S_x + iS_y|\mathbf{q},t\rangle$ quantum average, at any state

$$|t,\mathbf{q}\rangle = \cos\frac{\beta}{2}|0,0\rangle - e^{-i\mathcal{E}_q t}\sin\frac{\beta}{2}|1,\mathbf{q}\rangle$$

we come to a zero result: $\langle\mathbf{q},t|\widehat{S}_x + i\widehat{S}_y|t,\mathbf{q}\rangle \equiv 0.$

$\mathcal{E}_q = \epsilon_Z + q^2/2M_\mathrm{X}$ is the spin exciton energy at small dimensionless **q**.)

Thus the time of the Kerr signal decay is equal to the transition time of zero exciton conversion into nonzero one with the same energy:

$$|1,0\rangle \Rightarrow |1,\mathbf{q}\rangle_{q\to 0}.$$

What kind of interaction is responsible for the stochastization?

The perturbation responsible for the $|1,0\rangle \Rightarrow |1,\mathbf{q}\rangle_{q\to 0}$. conversion must be:
(i) a spin non-conserving coupling changing the S, but not changing the S_z quantum numbers; and
(ii) violating the translation invariance.

The most likely candidate is a term corresponding to the spatial fluctuations of the *g*-factor in 2D electron gas, i.e., the Zeeman energy is actually

$$\epsilon_Z + g_1(\mathbf{r})\mu_B B,$$

and the perturbation Hamiltinian is

$$\widehat{V}_g = -\frac{1}{2}\mu_B B \sum_i \begin{pmatrix} g_1(\mathbf{r}_i) & 0 \\ 0 & -g_1(\mathbf{r}_i) \end{pmatrix}_i$$

In terms of the `excitonic representation' (secondary quantization) the Hamiltonian is

$$\widehat{V}_g = -\frac{1}{2} N_\phi \sum_{\mathbf{q}} \overline{g}_{1\mathbf{q}}\, \mu_B B (A_{\mathbf{q}} - B_{\mathbf{q}})$$

where $$g_1(\mathbf{r}) = \sum_{\mathbf{q}} \overline{g}_{1\mathbf{q}} e^{i\mathbf{qr}}$$ and

$$A^\dagger_{\mathbf{q}} = \frac{1}{\mathcal{N}_\phi} \sum_p e^{-iq_x p} a^\dagger_{p+\frac{q_y}{2}} a_{p-\frac{q_y}{2}} \quad , \qquad B_{\mathbf{q}} = A_{\mathbf{q}}(a \to b).$$

The `key point' is calculation of the matrix element

$$\mathcal{M}_{\mathbf{q}} = \langle 0|\mathcal{Q}_{\mathbf{q}}|\, \widehat{V}_g \,|\mathcal{Q}^\dagger_0|0\rangle,$$

then usual procedure

$$\frac{1}{\tau} = \frac{2\pi}{\hbar} \sum_{\mathbf{q}} |\mathcal{M}_{\mathbf{q}}|^2\, \delta(\epsilon_Z - \mathcal{E}_{\mathbf{q}})$$

Let us assume that the *g*-disorder is Gaussian and governed by correlator

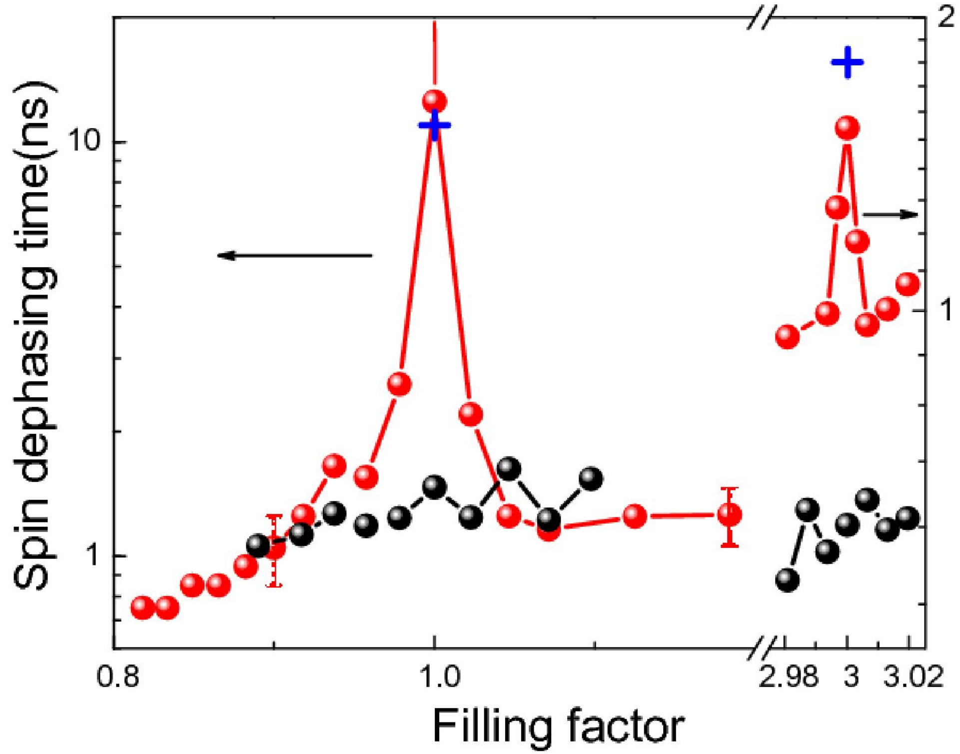
$$K(\mathbf{r}) = \int g_1(\mathbf{r}_0) g_1(\mathbf{r}_0 + \mathbf{r}) d\mathbf{r}_0 / A$$

parameterized by fluctuation amplitude Δ_g and correlation length Λ_g , so

$$K(\mathbf{r}) = \Delta_g^2 e^{-r^2/\Lambda_g^2}.$$

The result is

$$\frac{1}{\tau} = \pi M_{\mathrm{x}} (\mu_B B \Delta_g \Lambda_g)^2 / 2\hbar l_B^2.$$



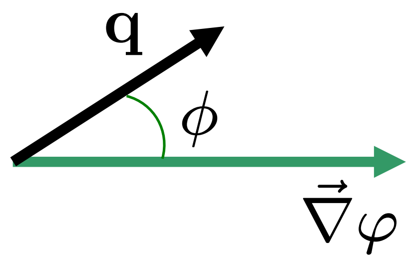
Some `hidden' details

Really argument of the δ–function in $\sum_{\mathbf{q}} |\mathcal{M}_{\mathbf{q}}|^2 \, \delta(\epsilon_Z - \mathcal{E}_{\mathbf{q}})$

is

$$\mathcal{E}_{\mathbf{q}} - \epsilon_Z = \frac{q^2}{2M_{\mathrm{X}}} - q|\vec{\nabla}\varphi| l_B \sin\phi$$

where $\mathbf{q} = (q, \phi)$:



$0 < \phi < \pi$!!

$\vec{\nabla}\varphi$ is smooth random electrostatic field

$$\sum_{\mathbf{q}} |\mathcal{M}_{\mathbf{q}}|^2 \delta(\mathcal{E}_{\mathrm{q}} - \epsilon_Z)$$

$$= \frac{N_\phi}{2\pi} \int_0^\infty q\,dq \int_0^\pi d\phi |\mathcal{M}_{\mathbf{q}}|^2 \, \delta\left(q^2/2M_{\mathrm{X}} - q|\vec{\nabla}\varphi| l_B\right) \sin\phi$$

$$\equiv \frac{N_\phi}{2\pi} \int_0^\infty dq \int_0^\pi d\phi |\mathcal{M}_{\mathbf{q}}|^2 2M_{\mathrm{X}} \delta\left(q - 2M_{\mathrm{X}}|\vec{\nabla}\varphi| l_B\right) \sin\phi$$

The result does not depend on $\vec{\nabla}\varphi$ but factor ½ appears due to $0 < \phi < \pi$.