GEOMETRIC RESPONSE AT THE EDGE

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Geometric aspects of the Quantum Hall Effect
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OUTLINE

I. Edge information from the effective theory

a. 1+1 edge

b. 0+2 domain wall. Remarks on ``bulk CFT"

II. Measurement of the *Shift* in photonic integer QH system (VERY briefly)

Some references:

1506.07171 with Abanov, Jensen

1511.07381 with Schine, Ryou, Sommer, Simon

GENERATING FUNCTIONAL

(NO) INTRODUCTION

$$W[A,g] = -i \ln \int D\psi \exp\Big(- iS[\psi; A, g]\Big)$$

Metric g describes non-relativistic geometry

$$g \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & g_{11}(x_i,t) & g_{12}(x_i,t) \\ 0 & g_{21}(x_i,t) & g_{22}(x_i,t) \end{bmatrix}$$

Can be represented via vielbeins.

$$e^0_\mu = (1,0,0) \qquad e^A_\mu = (0, e^A_i)$$

This representation leaves $SO(2)$ redundancy. Not Lorentz !

Spin connection: $$\omega_\mu = -\frac{1}{2}\delta^{AB} E^\nu_A D_\mu e_{\nu,B}$$

GENERATING FUNCTIONAL (1+1D BOUNDARY)

For FQH systems dimensionless terms are

$$W[A,g] = \frac{\nu}{4\pi}\int AdA + 2\bar{s}Ad\omega + \overline{s^2}\omega d\omega + \frac{c}{96\pi}\int Tr\left(\Gamma d\Gamma + \frac{2}{3}\Gamma^3\right)$$

We place $W[A,g]$ on a manifold with a boundary.

Then $U(1)$, $SO(2)$ and $Diff$ invariance is lost.

If the non-invariance can be compensated by local boundary terms then there are no (gapless) degrees of freedom induced the boundary

If the non-invariance cannot be compensated by local boundary terms then there are (gapless) degrees of freedom induced the boundary

$$Z_{full} = Z_{bulk} \cdot Z_{edge}$$

EXTRINSIC CURVATURE

Extrinsic curvature one-form

$$K = n^i D t_i$$

K differs from ω evaluated at the boundary by a frame rotation

$\omega + K = d\theta$ $\longrightarrow$ Gauss-Bonnet theorem

$$\frac{1}{2\pi}\int_M d\omega + \frac{1}{2\pi}\int_{\partial M} K = \chi$$

WEN-ZEE TERM AND BOUNDARY

The following combination is invariant

$$W_{WZ} = \frac{\nu\bar{s}}{2\pi}\int_M A d\omega + \frac{\nu\bar{s}}{2\pi}\int_{\partial M} A \wedge K$$

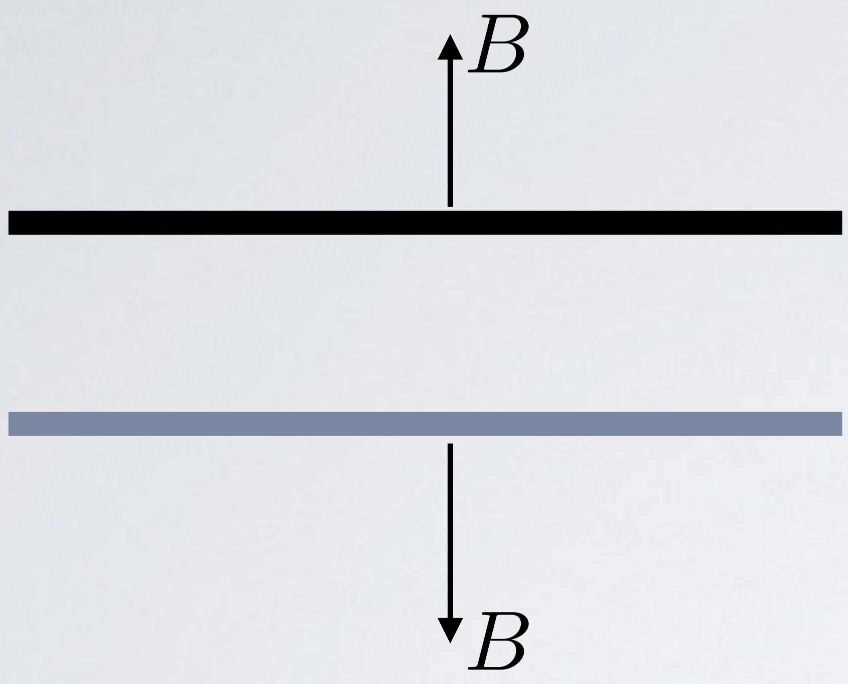
This slightly generalizes the notion of shift

$$N = \nu N_\phi + \nu\bar{s}\chi$$

Similarly we improve the second Wen-Zee term

$$W^{(2)}_{WZ} = \frac{\nu\overline{s^2}}{4\pi}\int_M \omega d\omega + \frac{\nu\overline{s^2}}{4\pi}\int_{\partial M} \omega \wedge K$$

EXAMPLE



IQH 1st LL filled

(parity conjugate) IQH 2nd LL filled

The edge modes can be completely gapped out

Is the resulting state trivial ? No $\bar{s} = 3/2 - 1/2 = 1$

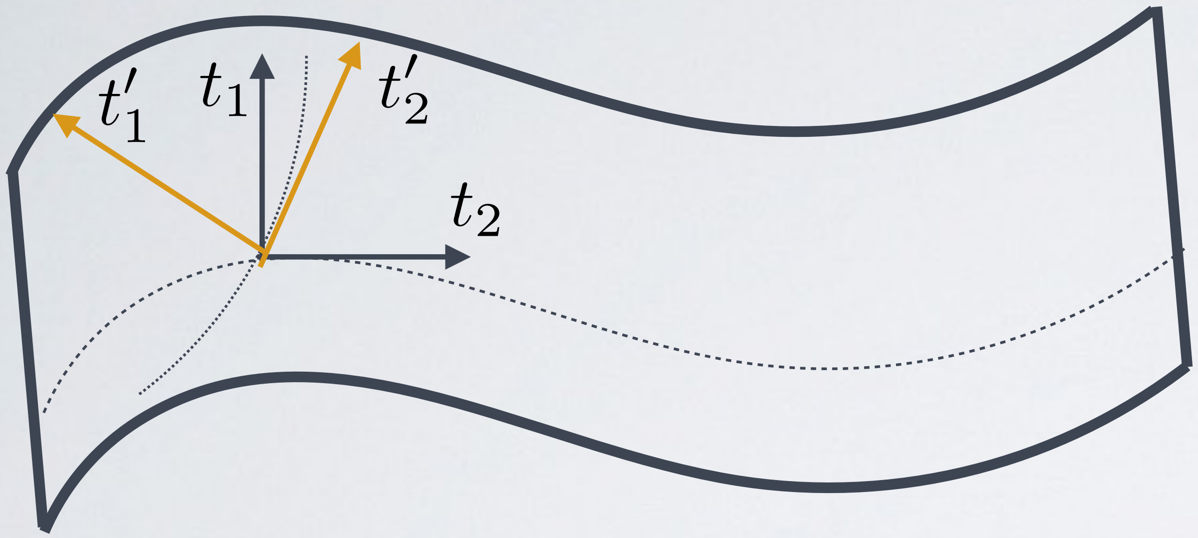
The generating functional has only Wen-Zee terms

$$W = \frac{1}{\pi} \int A d\omega + \frac{3}{4\pi} \int \omega d\omega$$

0+2 BOUNDARY

0+2 BOUNDARY

fixed time slice



$$K_\alpha = \delta^{AB} t^A_\beta D_\alpha t^{\beta,B}$$

The extrinsic curvature form is a connection under internal rotations

``Improvement terms'' do not cancel the anomaly, but shift it from frame rotations to intrinsic rotations

$$W[A,g] = \frac{\nu}{4\pi}\int AdA + 2\bar{s}Ad\omega + \overline{s^2}\omega d\omega + \frac{c}{96\pi}\int Tr\left(\Gamma d\Gamma + \frac{2}{3}\Gamma^3\right)$$

Mixed anomaly?!

ANOMALIES

Schematically the violated conservation laws are

$$D_i J^i = \frac{\nu}{2\pi} B + \frac{\nu \bar{s}}{2\pi} R \qquad D_j T^{ji} = \frac{\nu \bar{s}}{2\pi} \epsilon^{ij} \partial_j B + \frac{c - 12\nu \overline{s^2}}{48\pi} \epsilon^{ij} \partial_i R$$

There are no mixed anomalies! Can be removed by redefinition of operators. To do this we re-write the generating functional

$$W = \frac{\nu}{4\pi} \int \Big(A + \bar{s}\omega\Big) d \Big(A + \bar{s}\omega\Big) - \frac{1}{48\pi} \Big[c - 12\nu \, \mathrm{var}(s)\Big] \int \omega d\omega$$

We used

$$\mathrm{Tr} \left(\Gamma d\Gamma + \frac{2}{3} \Gamma^3 \right) = -2\omega d\omega + \frac{1}{3} (edE)^3$$

And defined

$$\mathrm{var}(s) = \overline{s^2} - \bar{s}^2$$

ANOMALIES II

$$D_i J^i = \frac{\nu}{2\pi} B \qquad\qquad D_j T^{ji} = \frac{\tilde{c}}{48\pi} \epsilon^{ij} \partial_j R$$

$$J = \frac{\delta W}{\delta(A + \bar{s}\omega)} \qquad\qquad \tilde{c} = c - 12\nu \mathrm{var}(s)$$

In the most general case bulk and edge ``gravitational anomalies'' are <u>different</u>.

When a FQH state comes from a CFT (Laughlin, Pfaffian, Read-Rezayi) then

$$\mathrm{v}ar(s) = 0$$

Bradlyn Read

and anomalies agree. There is ``bulk-boundary correspondence''.

When an FQH state does not come from a CFT (as a single conformal block) anomalies do not agree.

RELEVANT OBSERVABLE

An observable that knows about the anomalies is Odd (Hall) viscosity

$$\eta_H = \frac{\bar{s}}{2}\rho + \frac{\tilde{c}}{24}\frac{R}{4\pi}$$

It is better to look at ``viscositance''

Not c !

$$\bar{\eta}_H = \frac{1}{V}\int \eta_H = \frac{\bar{s}}{2}\bar{\rho} + \frac{\tilde{c}}{24}\frac{\chi}{V}$$

$$\bar{\rho} = \frac{N}{V}$$ average particle number

``finite size correction''

cannot be seen on torus

EXPLICIT CONSTRUCTION OF BOUNDARY THEORY

Start with Chern-Simons effective theory on $S^2 \times \mathbb{R}$

$$S_{eff} = \frac{k}{4\pi}\int ada - \frac{1}{2\pi}\int \bar{A}da - \frac{s}{2\pi}\int \bar{\omega}da$$

$$\nabla \times \bar{A} = B \qquad\qquad \nabla \times \bar{\omega} = R$$

At 1+1 boundary there is a gapless theory

$$S_{edge} = \frac{k}{4\pi}\int \partial_x\phi\partial_0\phi - v\partial_x\phi\partial_x\phi$$

Meaning

$$\int Dae^{-iS_{eff}[a;A,g]} = e^{-iW[A,g]}\int D\phi e^{-iS_{edge}[\phi;A,g]}$$

EXPLICIT CONSTRUCTION OF ``BULK'' THEORY

At 0+2 boundary there is a theory

$$S_{bulk} = \frac{k}{4\pi}\int D_x\phi D_y\phi - vD_x\phi D_x\phi - \frac{2}{k}B\phi - 2\frac{s}{k}R\phi$$

Shift the variable

$$\partial\varphi = \partial\phi + \frac{1}{k}\bar{A} + \frac{s}{k}\bar{\omega}$$

$$S_{bulk} = \frac{k}{4\pi}\int \partial_x\varphi\partial_y\varphi - v\partial_x\varphi\partial_x\varphi - \frac{2}{k}\bar{A}\wedge d\varphi - 2\frac{s}{k}\bar{\omega}\wedge d\varphi + (anomaly)$$

Double the theory, integrate by parts, anomalies cancel

$$S_{bulk} = \frac{k}{\pi}\int \partial\varphi\bar{\partial}\varphi + i\frac{1}{2k}B\varphi + i\frac{s}{4k}R\varphi = \frac{k}{\pi}\int \partial\varphi\bar{\partial}\varphi - i\beta\int \rho_0\varphi$$

MULTICOMPONENT STATES

$$S_{eff} = -\frac{1}{4\pi}\int \left[K_{IJ}a^I da^J + 2q_I A da^I + 2s_I \omega da^I\right]$$

Identical arguments lead to

$$S_{bulk} = \frac{1}{4\pi}\int K_{IJ}\partial_x\varphi_I\partial_y\varphi_J - V_{IJ}\partial_x\varphi_I\partial_x\varphi_J - \frac{i}{2\pi}q_I\bar{A}\wedge d\varphi_I - \frac{i}{2\pi}s_I\bar{\omega}\wedge d\varphi_I + (\text{anomaly})$$

Doubling

$$S_{bulk} = \frac{1}{\pi}\int K_{IJ}\partial\varphi_I\bar{\partial}\varphi_J - V_{IJ}\partial_x\varphi_I\partial_x\varphi_J - 2i\Big(q_I B + s_I R\Big)\wedge d\varphi_I$$

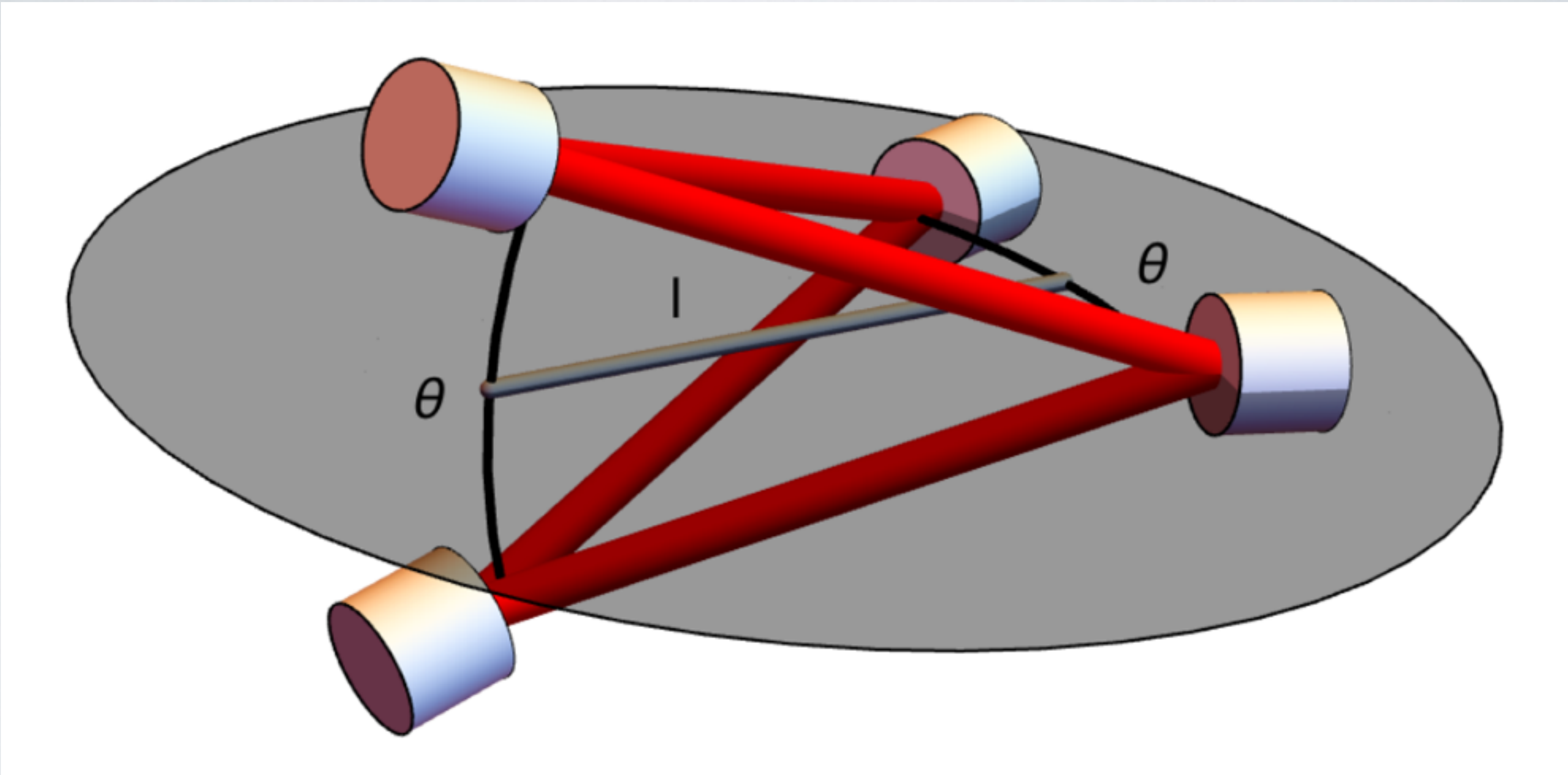
Every field φ_I has it's own background charge s_I

Cannot remove all of them by an operator redefinition.

$$c \longleftrightarrow \tilde{c}$$

MEASUREMENT OF SHIFT

(On a cone)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DENSITY OF STATES

$\nu = 1$

$$\rho = \frac{B}{2\pi} + \bar{s}\frac{R}{4\pi}$$

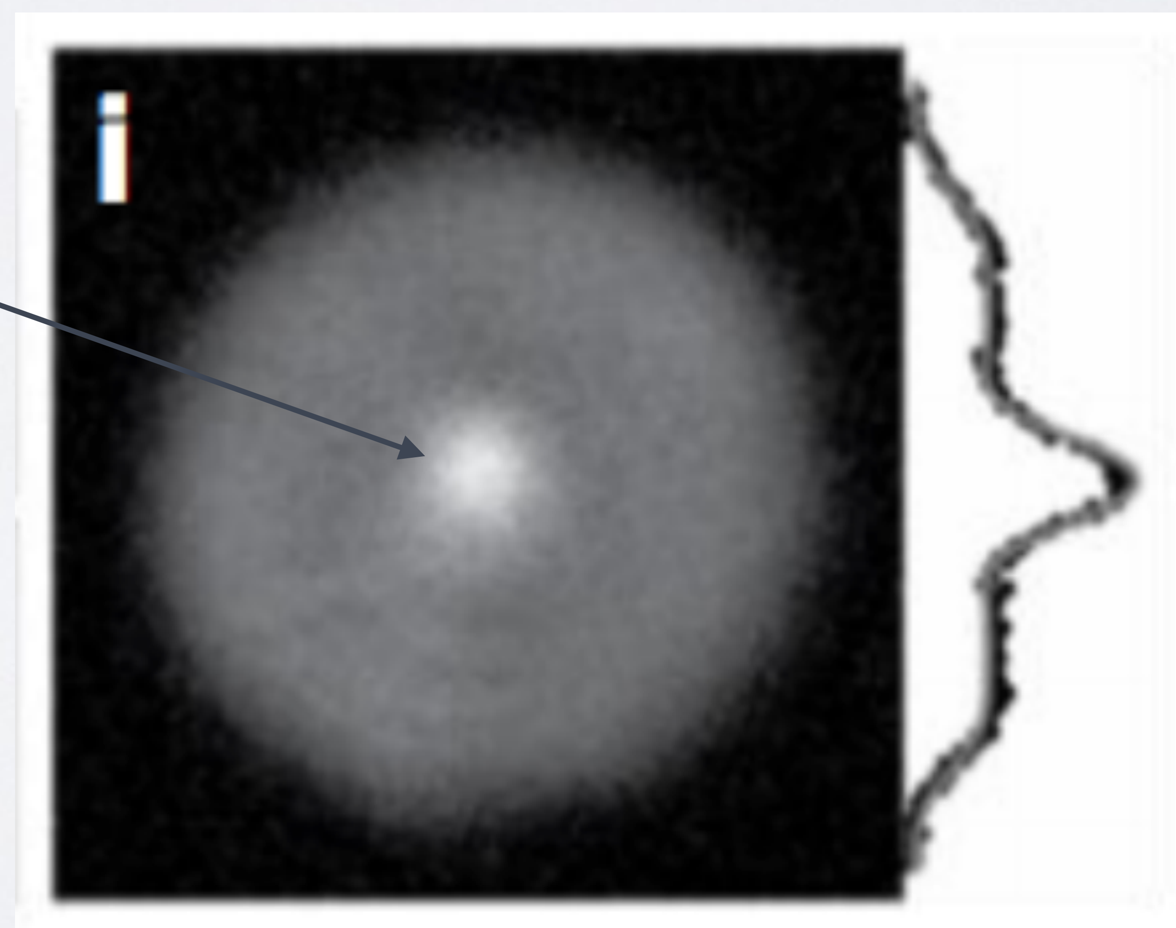
$$R = 4\pi\Big(1 - \frac{1}{m}\Big)\delta(z)$$

$$\bar{\rho} = \frac{B}{2\pi}$$

$$\delta N = \int \rho - \bar{\rho} = \bar{s}\left(1 - \frac{1}{m}\right)$$

There should be (fractional) excess of states

Extra $1/3$ state



CONCLUSIONS

I QH is a non-relativistic system. Effective theory must reflect that.

II Wen-Zee terms do not lead to edge states, however they define a bulk topological invariant $\bar{s}$

III ``Bulk CFT'' can be *derived* from Chern-Simons theory

IV Generally, anomalies of bulk and edge theories are *different*

V When a QH state is constructed a single conformal block the anomalies coincide.

VI For Jain states the anomalies are *different*

VII $\bar{s}$ has been measured for IQHE! Perhaps c is next?