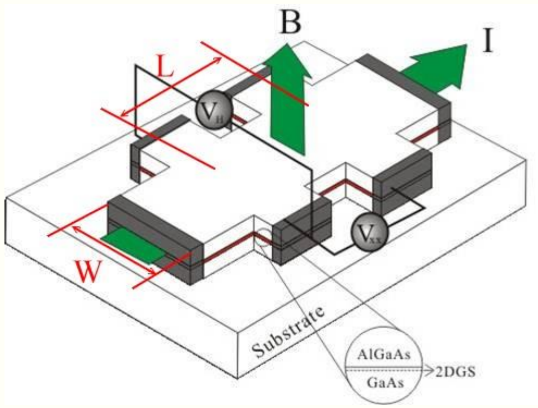
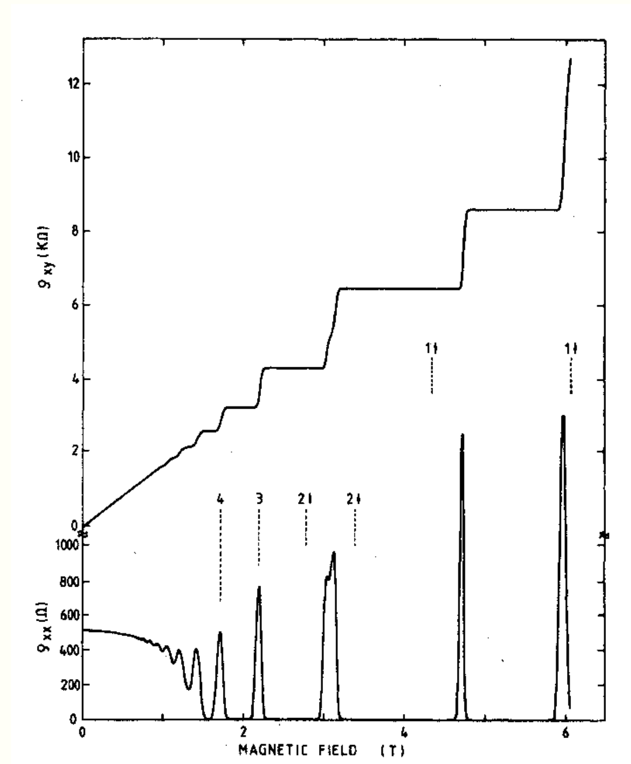
Geometric disorder and critical behavior at the integer quantum Hall plateau transition

Ilya A. Gruzberg
Ohio State University

Coauthors: A. Sedrakyan, Yerevan
A. Klümper, Wuppertal
W. Nuding, Wuppertal

Integer quantum Hall effect





K. v. Klitzing, Rev. Mod. Phys. **56** (1986)

- Two-dimensional electron gas
- Strong magnetic field, low temperature
- Hall resistance shows plateaus

$$R_H = \frac{1}{n}\frac{h}{e^2}$$

- Transitions between the plateaus – example of an Anderson transition
- Critical properties near Anderson transitions – largely unsolved problem

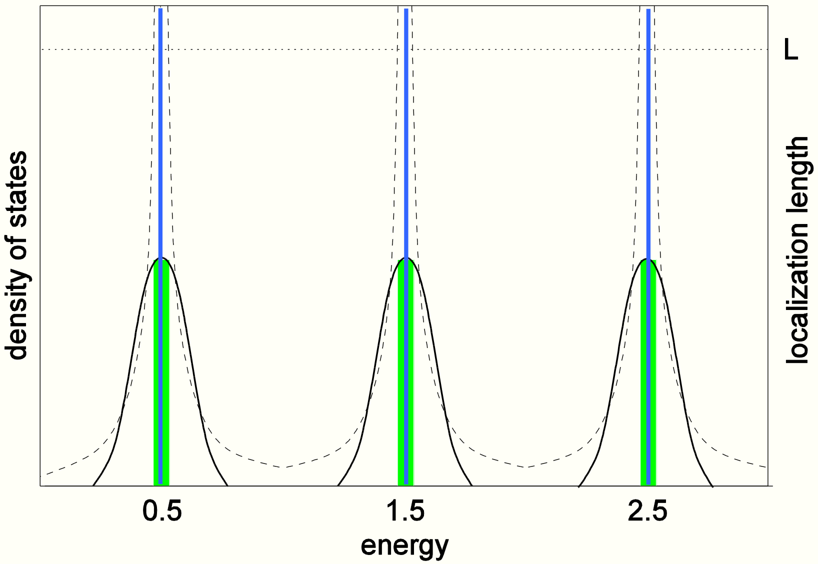
IQH and localization in strong magnetic field

- Single electron in a magnetic field and a random potential

$$H = \frac{1}{2m}\Big(-i\hbar\nabla + \frac{e}{c}\mathbf{A}\Big)^2 + U(\mathbf{r})$$

- Without disorder: Landau levels

$$E_n = \Big(n + \frac{1}{2}\Big)\hbar\omega_c, \quad \omega_c = \frac{eB}{mc}$$



- Disorder broadens the levels and localizes most states

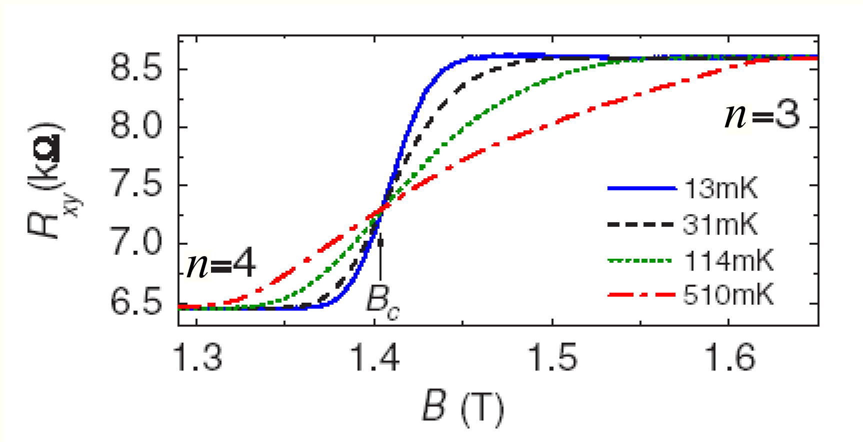
- Extended states near E_n (green)

- Transition between QH plateaus upon varying E_F or B

- Localization length diverges $\xi(E) \propto |E - E_n|^{-\nu}$

Critical scaling near IQH plateau transition

- Recent experiments

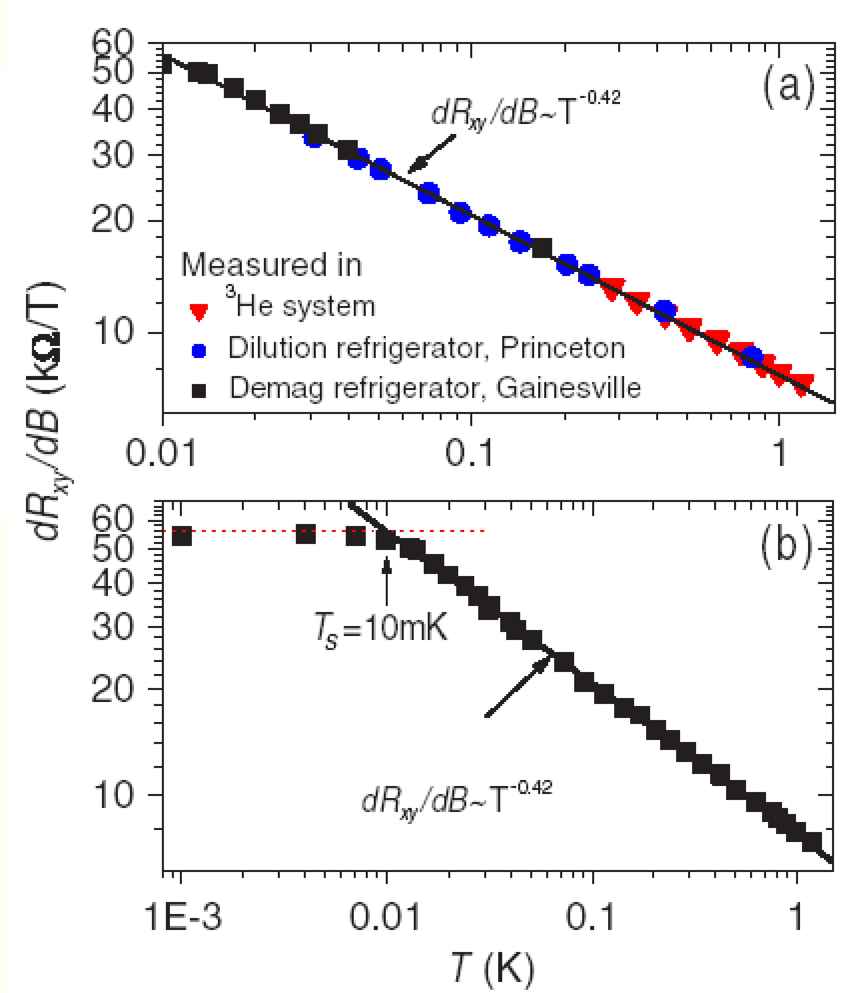


W. Li et al., PRL **94** (2005); **102** (2009)

$$\left.\frac{dR_{xy}}{dB}\right|_{B_c} \sim T^{-\kappa}$$

$$\kappa = \frac{1}{z\nu} \approx 0.42$$

$$\nu \approx 2.38, \quad z = 1$$



Methods and models for theory of IQH transition

- No small parameter, no perturbation theory

- Recent analytical approaches include

  - Field theory: non-linear sigma model, symmetry analysis and CFT

    IAG, A. W. W. Ludwig, A. D. Mirlin, M. R. Zirnbauer `11-13
    R. Bondesan, D. Wieczorek, M. R. Zirnbauer `14
    R. Bondesan, M. R. Zirnbauer, work in progress

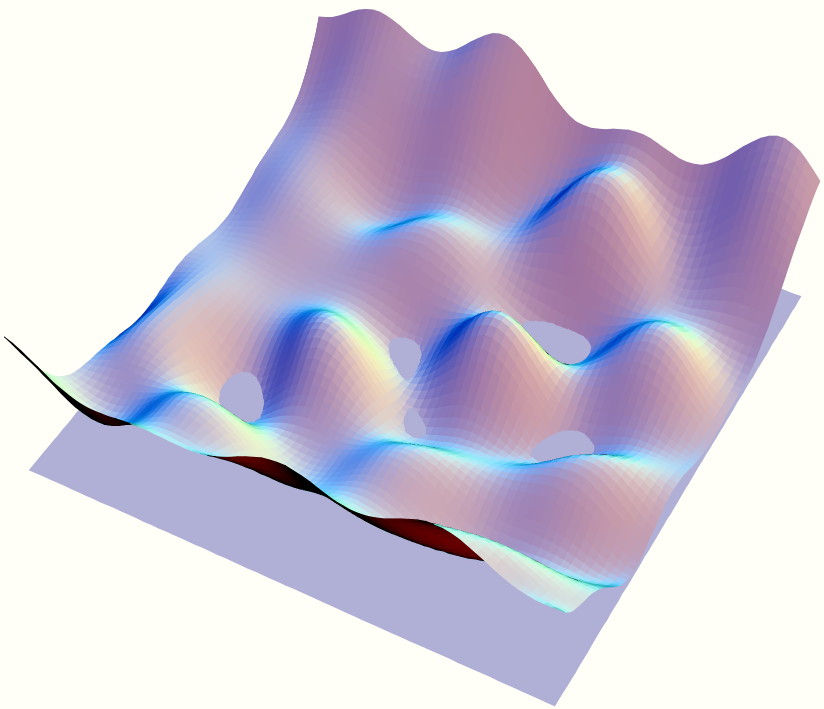
  - Stochastic conformal geometry: SLE and conformal restriction; statistical mechanics and CFT

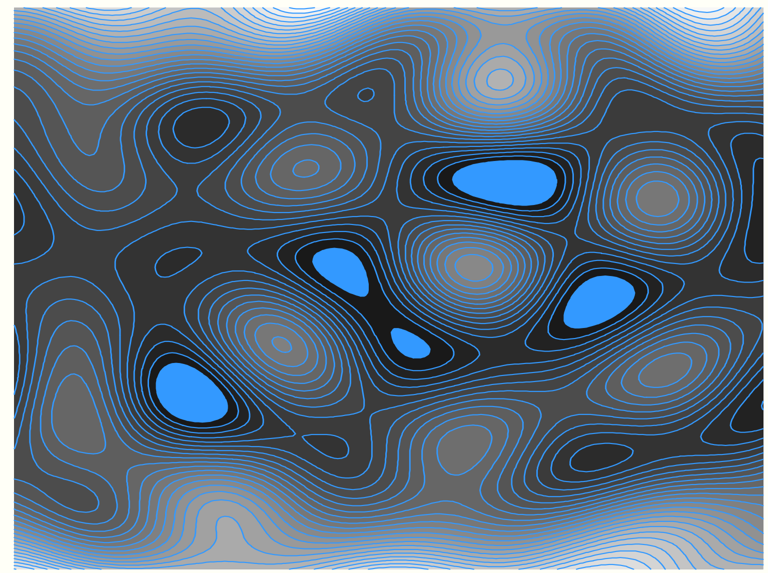
    E. Bettelheim, IAG, A. W. W. Ludwig `12
    E. Bettelheim, work in progress

- A lot of intuition comes from a network model, also numerics

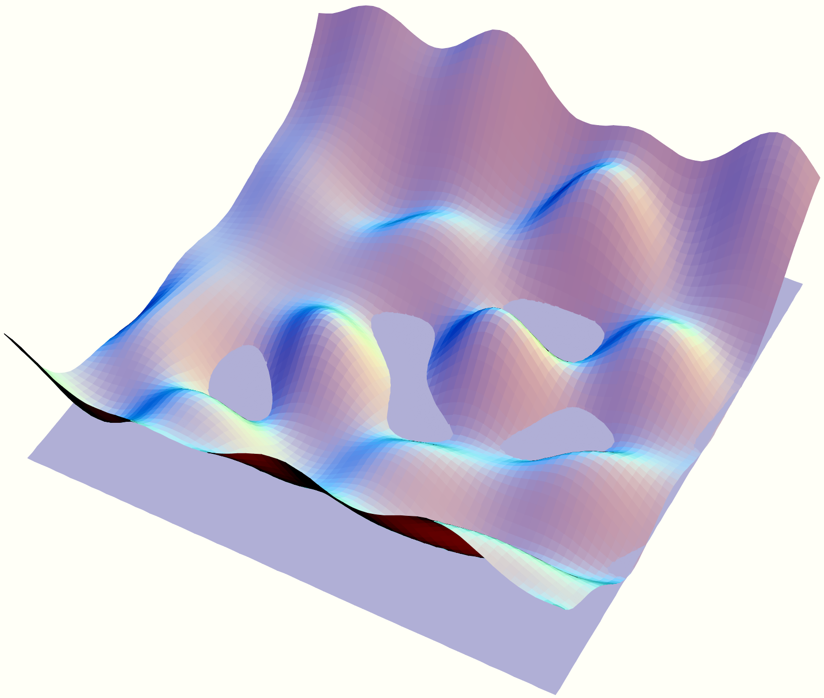
  J. T. Chalker and P. D. Coddington `88

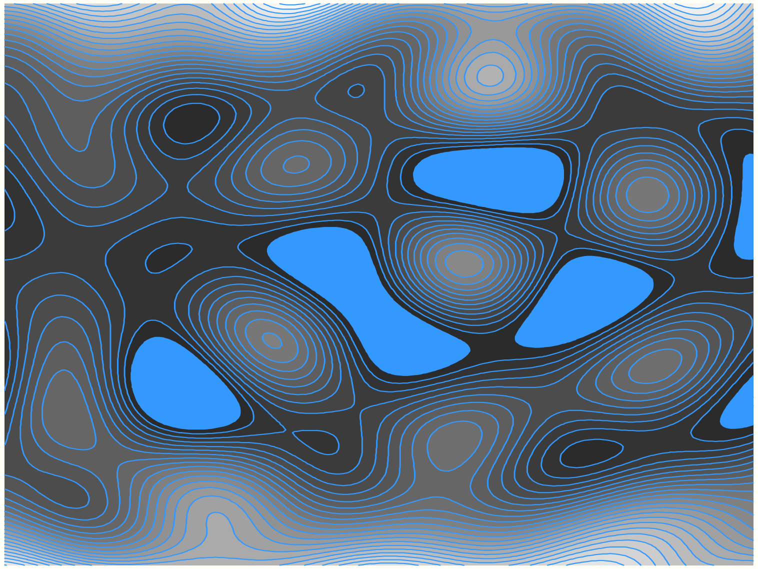
“Derivation” of the network model: Electrons in a smooth random potential



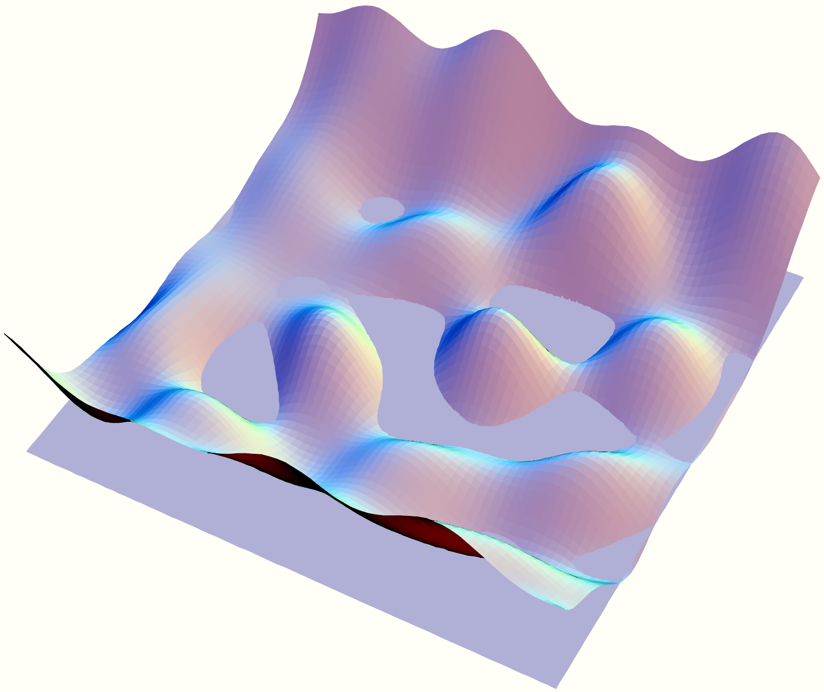


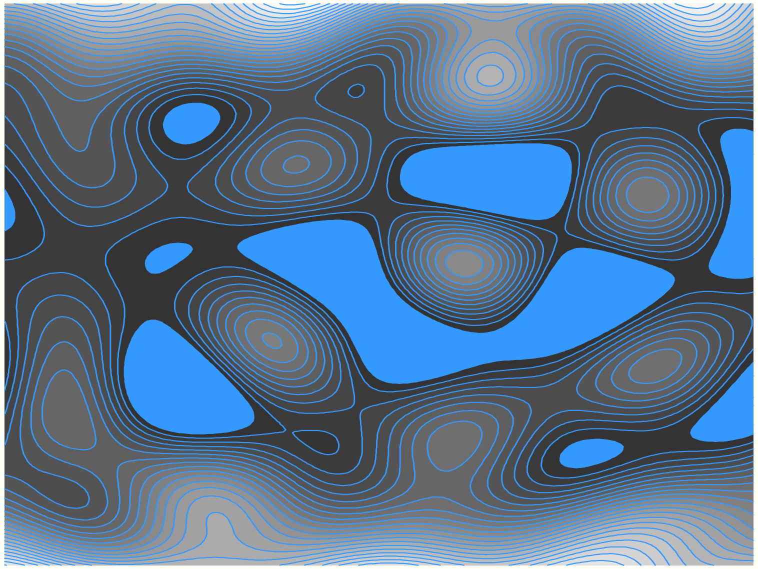
Electrons in smooth random potential



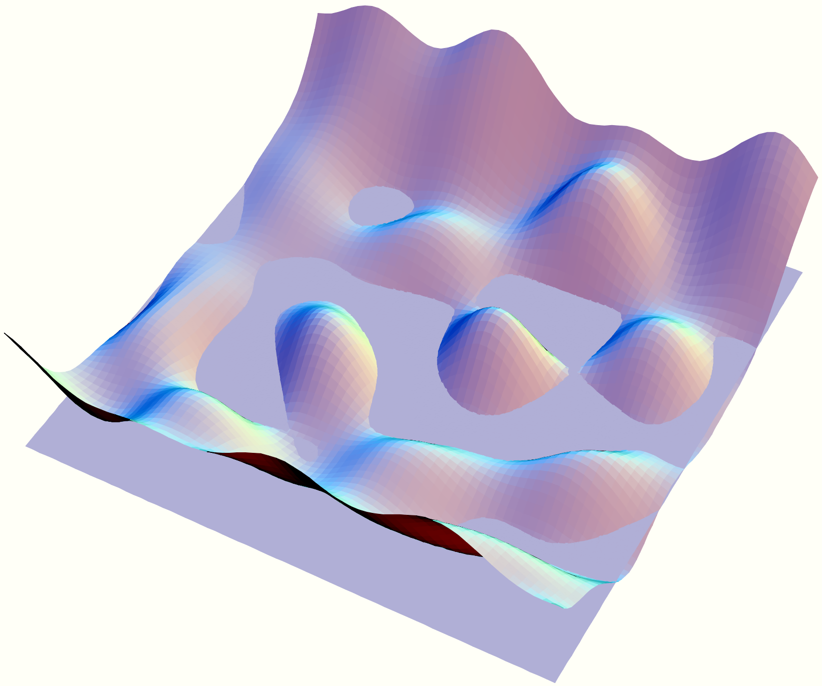


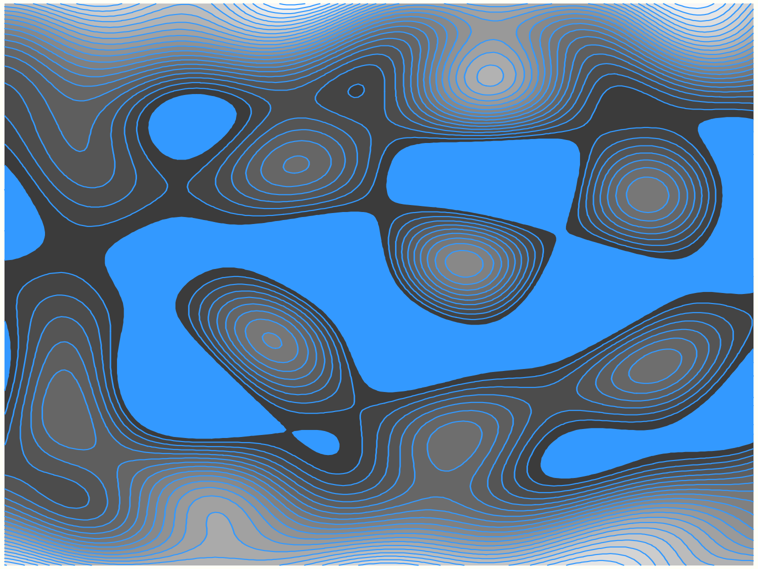
Electrons in smooth random potential



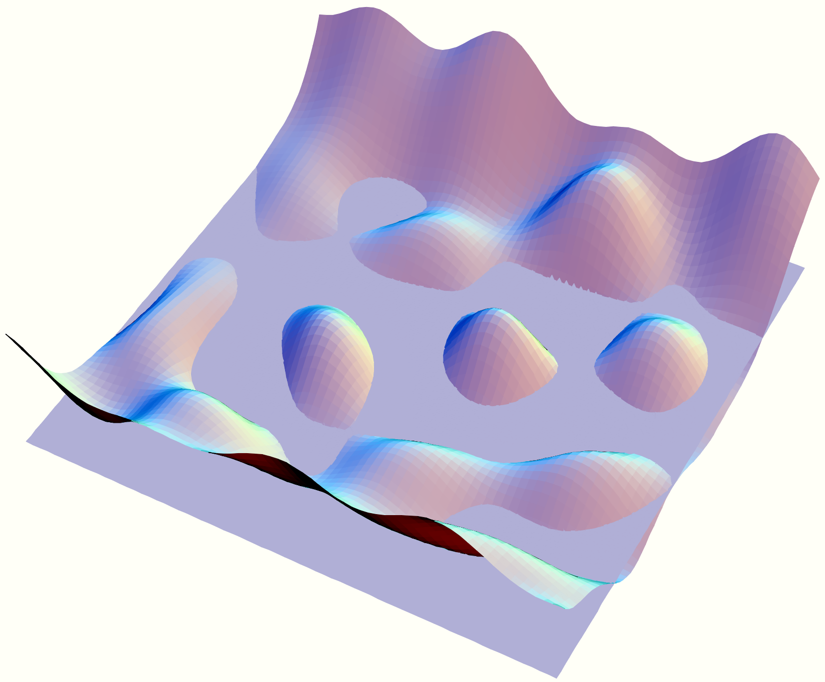


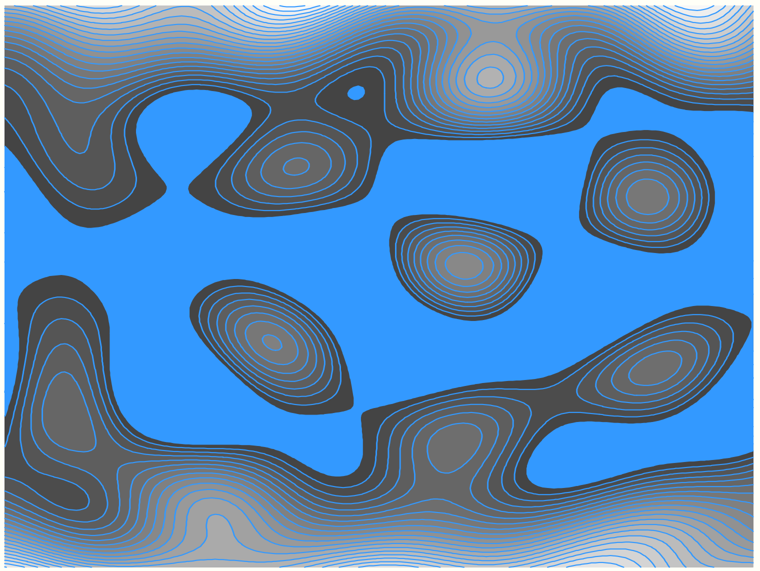
Electrons in smooth random potential



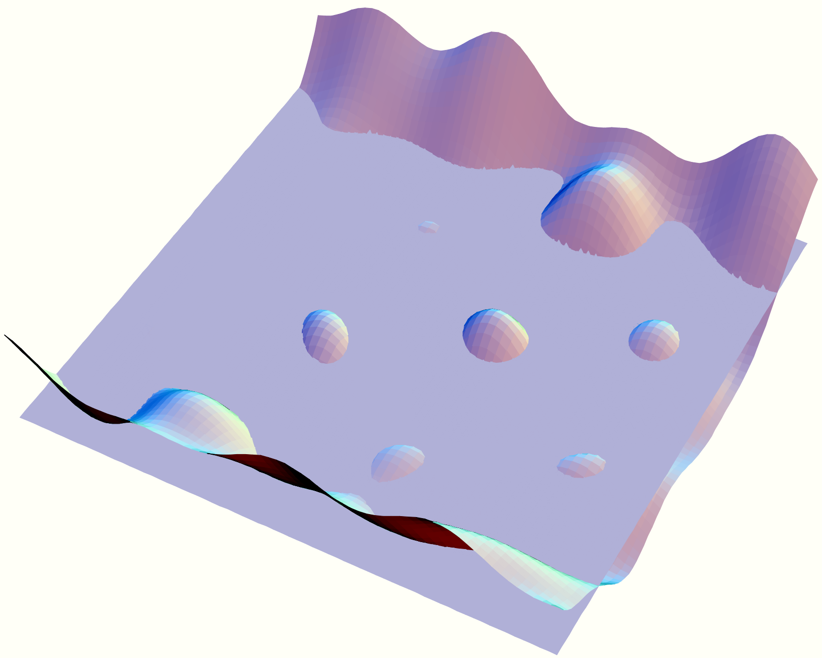


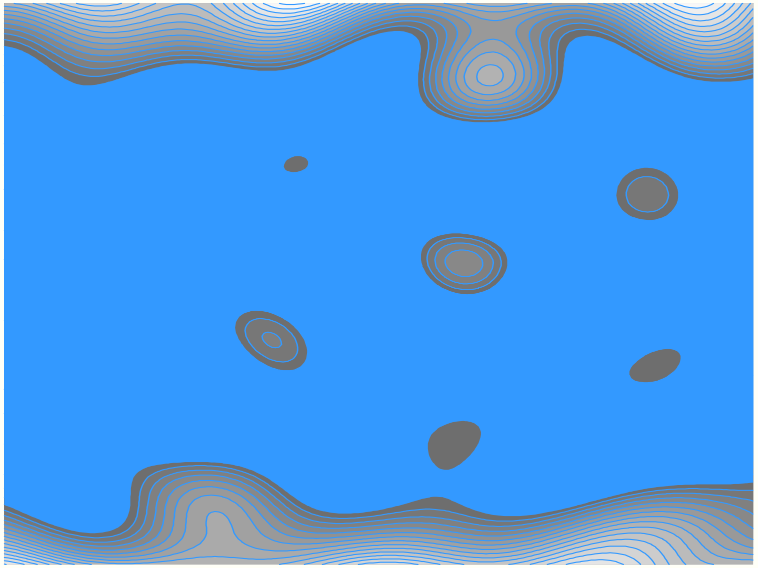
Electrons in smooth random potential





Electrons in smooth random potential





- The picture resembles classical percolation
- Essential difference:

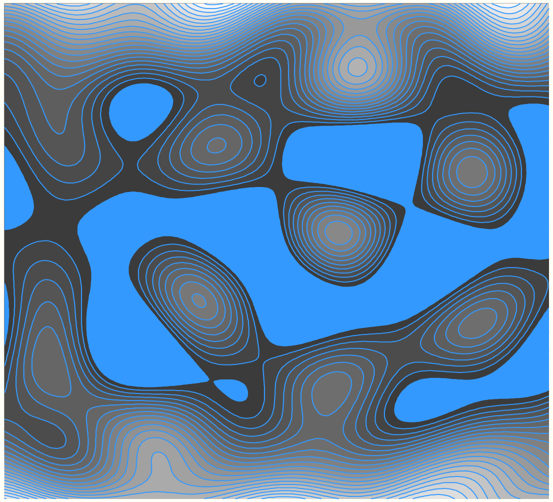
  - tunneling across saddle points

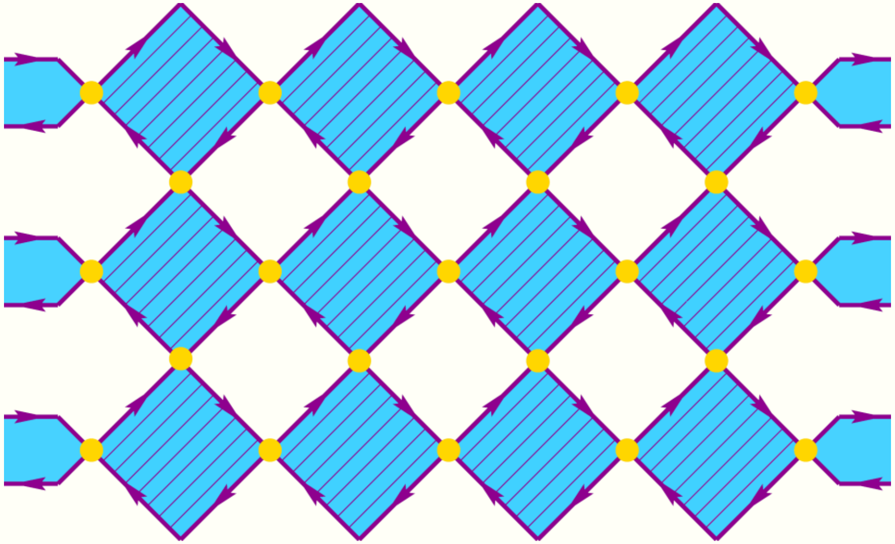
  - quantum interference and random Aharonov-Bohm phases

Chalker-Coddington network model

J. T. Chalker, P. D. Coddington `88

- Obtained from semi-classical drifting orbits in smooth potential

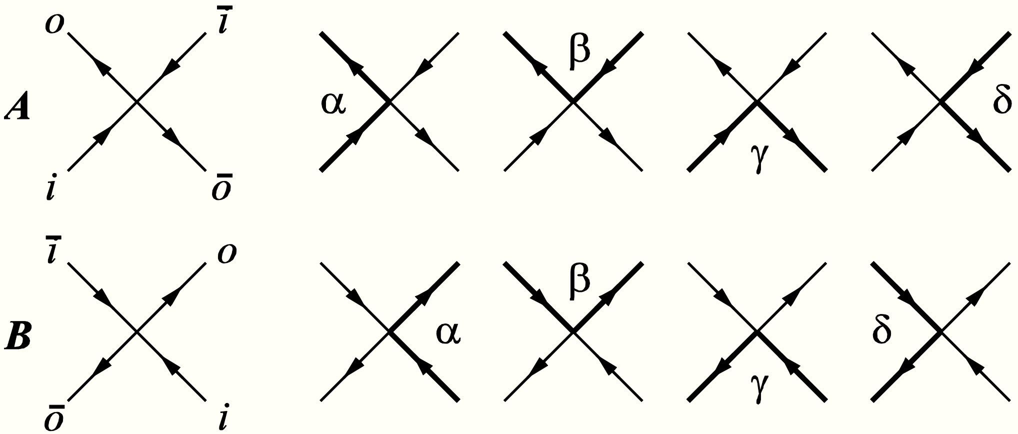




- Complex fluxes (currents) on links, scattering at nodes
- Regular lattice is convenient for numerical transfer matrix calculations

Chalker-Coddington network model

- States of the system specified by $Z \in \mathbb{C}^{N_l}$, N_l the number of links
- Evolution (discrete time) specified by a random $U \in \mathrm{U}(N_l)$

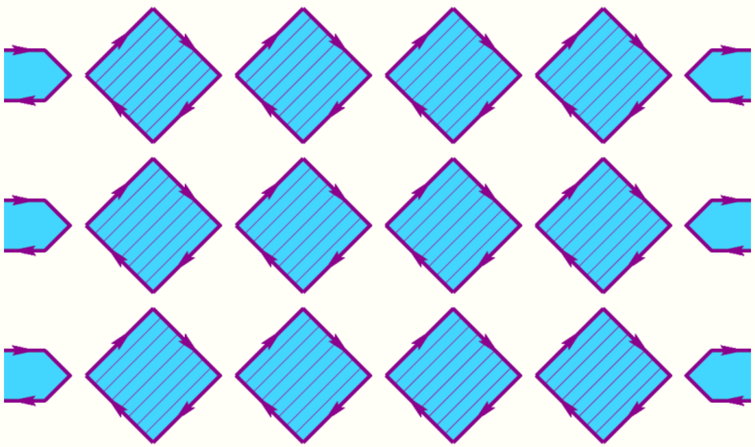


$$\begin{pmatrix} o \\ \bar{o} \end{pmatrix} = \mathcal{S} \begin{pmatrix} i \\ \bar{\imath} \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} i \\ \bar{\imath} \end{pmatrix}$$

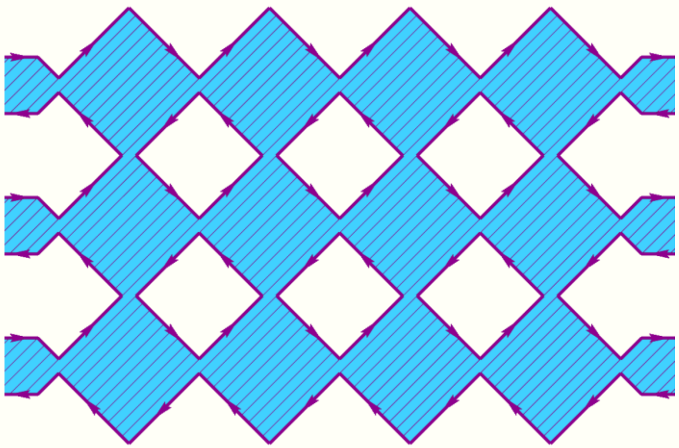
$$\mathcal{S}_S = \begin{pmatrix} e^{i\phi_1} & 0 \\ 0 & e^{i\phi_2} \end{pmatrix} \begin{pmatrix} \sqrt{1-t_S^2} & t_S \\ -t_S & \sqrt{1-t_S^2} \end{pmatrix} \begin{pmatrix} e^{i\phi_3} & 0 \\ 0 & e^{i\phi_4} \end{pmatrix}$$

Chalker-Coddington network model

- Extreme limits in the isotropic case (reminds percolation)



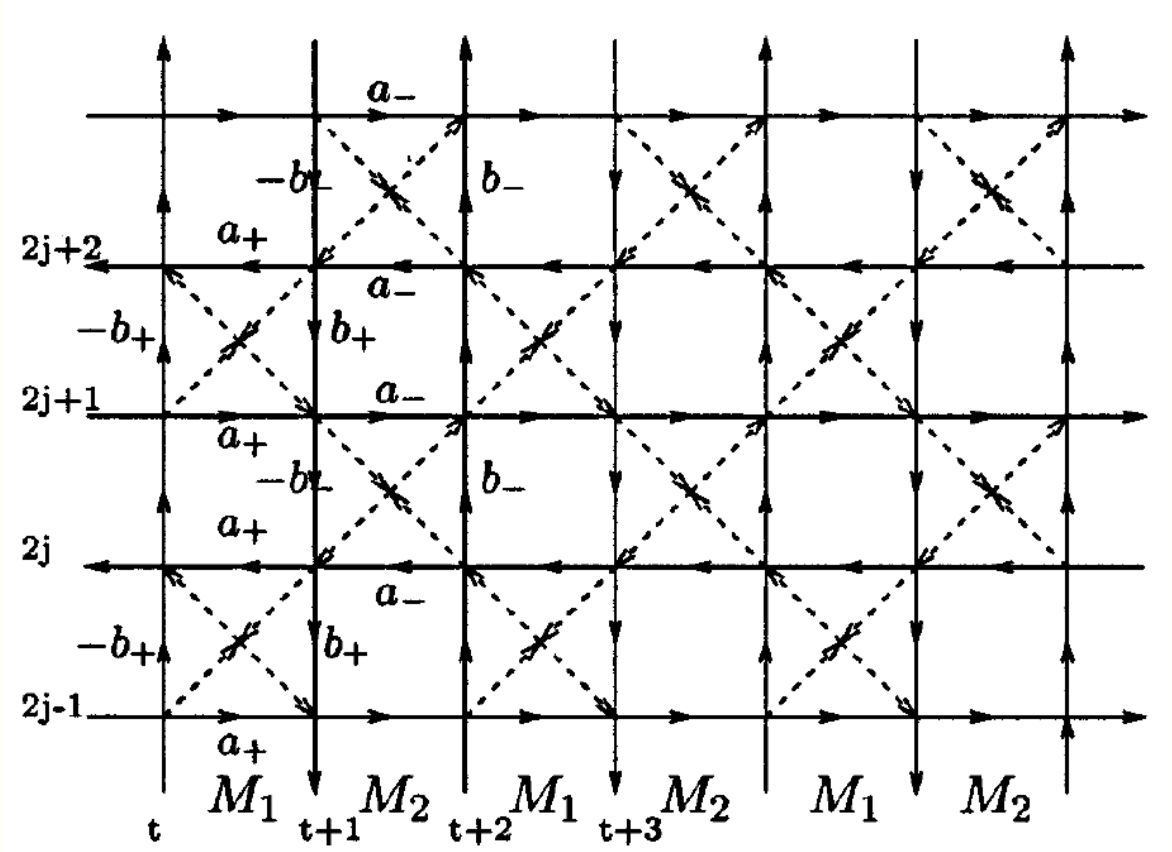
$t_A = 1$ Insulator



$t_A = 0$ Quantum Hall

- Critical point at $t_A^2 = 1/2$

Median Manhattan lattice



- R-matrices and action formulation

A. Sedrakyan \`03

Summary of numerical and experimental results

Table 3. Recent estimates of the critical exponent ν.

Slevin and Ohtsuki[1]	$2.593[2.587, 2.598]$	Obuse et al.[14]	$2.55 \pm .01$
Dahlhaus et al.[15]	$2.576 \pm .03$	Amado et al.[16]	$2.616 \pm .014$
Obuse et al '12	2.62 ± 0.06	Nuding et al '15	2.56

Table 1. Experimental values of critical exponents for the quantum Hall transition.

	$\kappa = 1/z\nu$	ν	z
Experiment of Li et al.[5] ($Al_xGa_{1-x}As$)	$0.42 \pm .01$	≈ 2.38	≈ 1
Experiment of Giesbers et al.[6] (graphene)	$0.41 \pm .04$	-	-

A puzzle

- All recent numerical results for ν agree with each other

$$\nu_{\mathrm{num}} \approx 2.6$$

- They all differ from the experimental value

$$\nu_{\mathrm{exp}} \approx 2.38$$

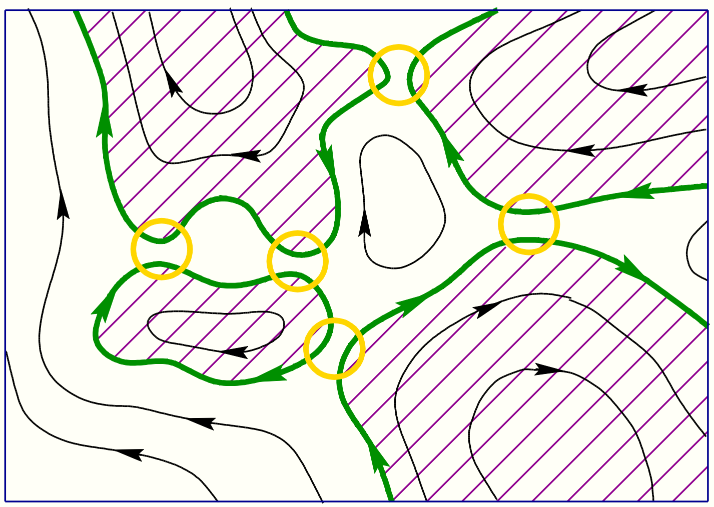
- Possible reason: electron-electron interactions

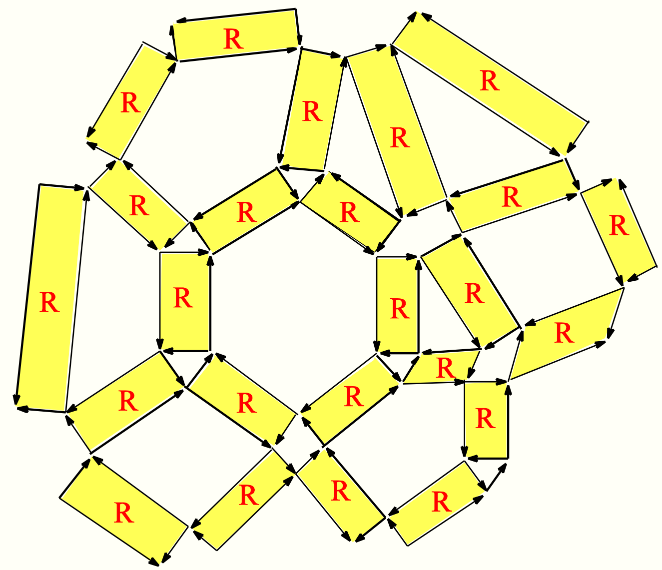
- We propose an alternative possible explanation:

Geometric disorder

Geometric disorder

- Arbitrary number of nodes around a puddle



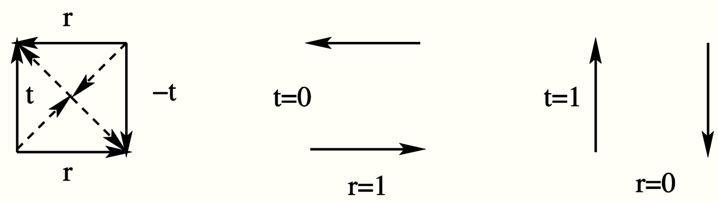


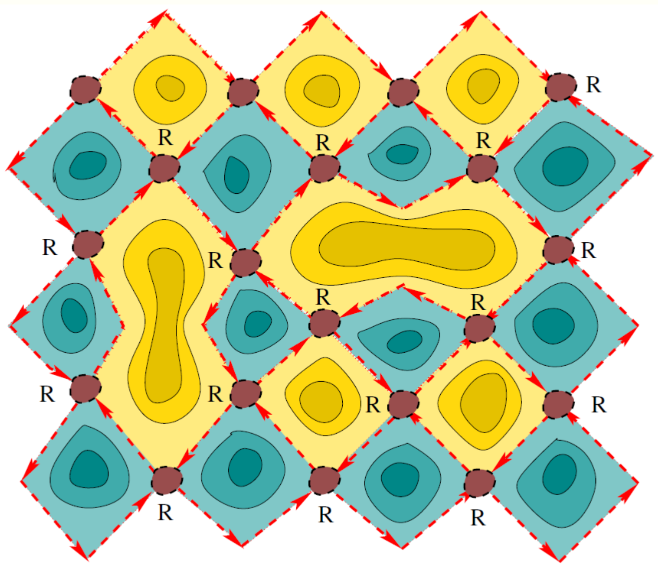
- What to do with this?

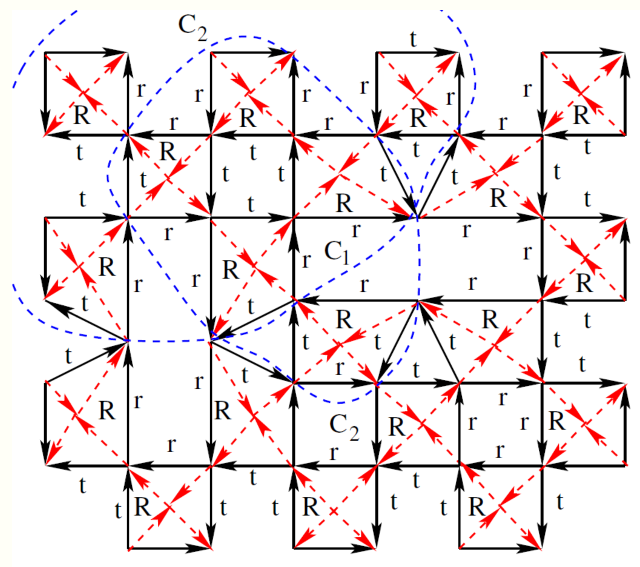
- How to implement numerically?

Geometric disorder: numerical implementation

- Randomly choose a node and open it up







- Result: $\nu = 2.37 \pm 0.02$

- Agrees with experiments, perhaps a coincidence

- Geometric disorder is relevant and needs to be understood

Continuum action for CC model: Dirac fermions

- Continuum limit of CC model

C. M. Ho, J. T. Chalker `96

$$S = \int d^2x\,\bar{\psi}[\sigma_\mu(i\overleftrightarrow{\partial}^\mu + A^\mu) + m\sigma_3 + V]\psi$$

- Random mass, gauge and scalar potentials $X = (A, m, V)$

- Quenched disorder, need supersymmetry (or replicas)

$$\overline{\langle O\rangle} = \int \mathcal{D}X \langle O\rangle = \int \mathcal{D}X \frac{\int \mathcal{D}\psi\, O(\psi)\, e^{-S[\psi,X]}}{\int \mathcal{D}\psi\, e^{-S[\psi,X]}}$$

$$= \int \mathcal{D}X \int \mathcal{D}\psi \mathcal{D}\phi\, O(\psi) e^{-S[\psi,X]-S[\phi,X]}$$

- Gaussian disorder induces quartic interactions between fermions and bosons

Continuum action for geometric fisorder: Dirac fermions coupled to quenched quantum gravity

- We propose that the continuum limit for the network with geometric disorder has additional coupling to quenched quantum gravity:

$$S = \int d^2x\, e\, \bar{\Psi}[\sigma_\mu e^\mu_\alpha (i \overleftrightarrow{\partial}^\alpha + A^\alpha) + m\sigma_3 + V]\Psi$$

- Disorder now includes geometric data (frames or metric)

$$X = (A, m, V, e)$$

- Needs further study

Summary

- Geometric disorder in the network model is a relevant perturbation and changes critical behavior
- Critical exponent $\nu = 2.37 \pm 0.02$, close to experimental value
- Possible field theory description using quenched quantum gravity
- More details in the afternoon talk by Ara Sedrakyan