One idea and four applications

in the Conformal Field Theory approach to QH wave functions

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Outline:

- **The context - Quantum Hall wave functions**
- **The CFT approach to QH wave functins**
- **The idea - how to represent quasielectrons by CFT**
- **The applications:**

1. **Jain wave functions and the HH-hierarchy**
2. **Hierarchy wave functions on a torus**
3. **Matrix Product State representation of quasielectrons**
4. **"Genuine" nonabelian hierarchies (M. Hermanns)**

Quantum Hall wave functions basics

1. The Laughlin wave functions

$$\Psi^L_m(z_1, \ldots z_N) = \prod_{i<j}(z_i - z_j)^m e^{-\frac{1}{4}\sum_i |z_i|^2}$$

2. The Jain wave functions

$$\Psi^J_{\frac{q}{2pq+1}}(\{\vec{r}_i\}) = \mathcal{P}_{LLL}\left[\prod_{i<j}(z_i - z_j)^{2p}\Phi_q(\{\vec{r}_i\})\right]$$

3. The Moore-Read wave function

$$\Psi^{MR}_{1/2}(\{\vec{r}_i\}) = \langle V_e(z_1)V_e(z_2)\ldots V_e(z_N)\mathcal{O}_{bg}\rangle$$

4. The Haldane-Halperin hierarchy wave functions

The MR Conformal Field Theory approach

Operators:

Electron operator: $V(z) = e^{i\sqrt{3}\varphi(z)}$

Hole operator: $H(\eta) = e^{\frac{i}{\sqrt{3}}\varphi(\eta)}$

$$\langle\varphi(z)\varphi(w)\rangle = -\ln(z-w)$$

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Wave functions:

$$\Psi_{\frac{1}{3}}(z_1,\dots z_N) = \prod_{i<j}(z_i - z_j)^3 e^{-\frac{1}{4}\sum_i |z_i|^2} = \langle V(z_i)\dots V(z_N)\rangle_{bg}$$

$$\Psi_{\frac{1}{3},1qh}(\eta; z_1,\dots z_N) = \prod_{i<j}(z_i - z_j)^3 \prod_i (z_i - \eta) e^{-\frac{1}{4}\sum_i |z_i|^2}$$

$$= \langle H(\eta)V(z_i)\dots V(z_N)\rangle_{bg}$$

Conformal Blocks

The MR Conformal Field Theory approach

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Conformal Blocks

$$= \langle H(\eta)V(z_i)\dots V(z_N)\rangle_{bg}$$

Insertion of $H(\eta_1)\dots H(\eta_M)$ **yields multi-quasihole wave functions!**

Are there similar expressions for the Laughlin quasielectron states?

Quasielectron wave functions:

The simplest choise $P(\eta) = e^{-\frac{i}{\sqrt{3}}\varphi(\eta)}$ **gives**

$$\Psi_L^{(qel)}(\bar{\eta}; z_1, \ldots, z_N) = e^{-\frac{1}{4m}|\eta|^2} \prod_i \frac{1}{z_i - \bar{\eta}} \prod_{i<j} (z_i - z_j)^m e^{-\frac{1}{4}\sum_i |z_i|^2}$$

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Laughlin's Quasielectron:

$$\Psi_L^{(qel)}(\bar{\eta}; z_1, \ldots, z_N) = e^{-\frac{1}{4m}|\eta|^2} \prod_i (2\partial_i - \bar{\eta}) \prod_{i<j} (z_i - z_j)^m e^{-\frac{1}{4}\sum_i |z_i|^2}$$

Jain's Quasielectron:

$$\Psi_J^{(qel)}(0; z_1, \ldots, z_N) = \sum_i (-1)^i \prod_{j<k}^{(i)} (z_j - z_k)^m \partial_i \prod_l^{(i)} (z_l - z_i)^{m-1} e^{-\frac{1}{4}\sum_i |z_i|^2}$$

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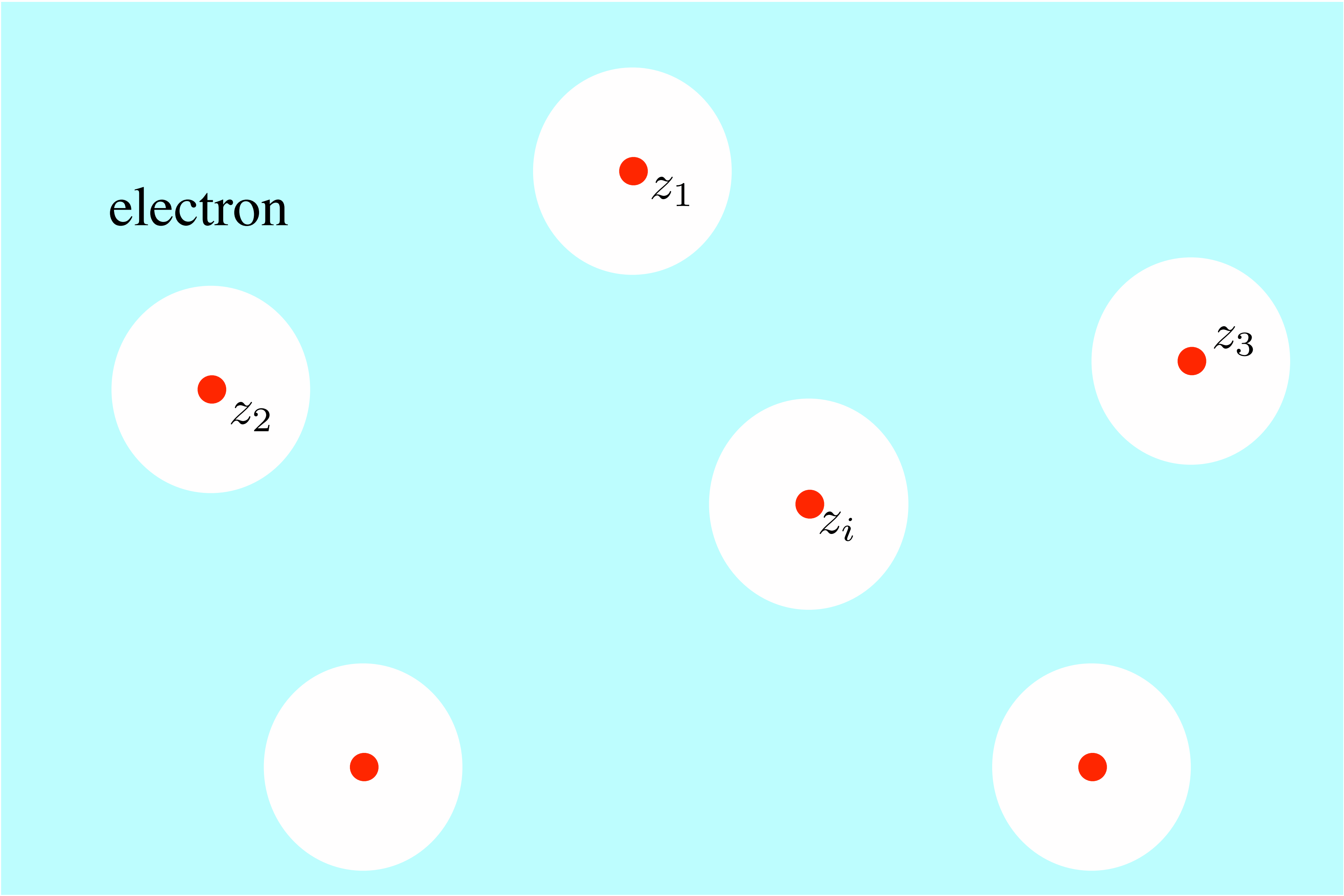
What about the connection to conformal field theory?

The idea: The QH quasielectron



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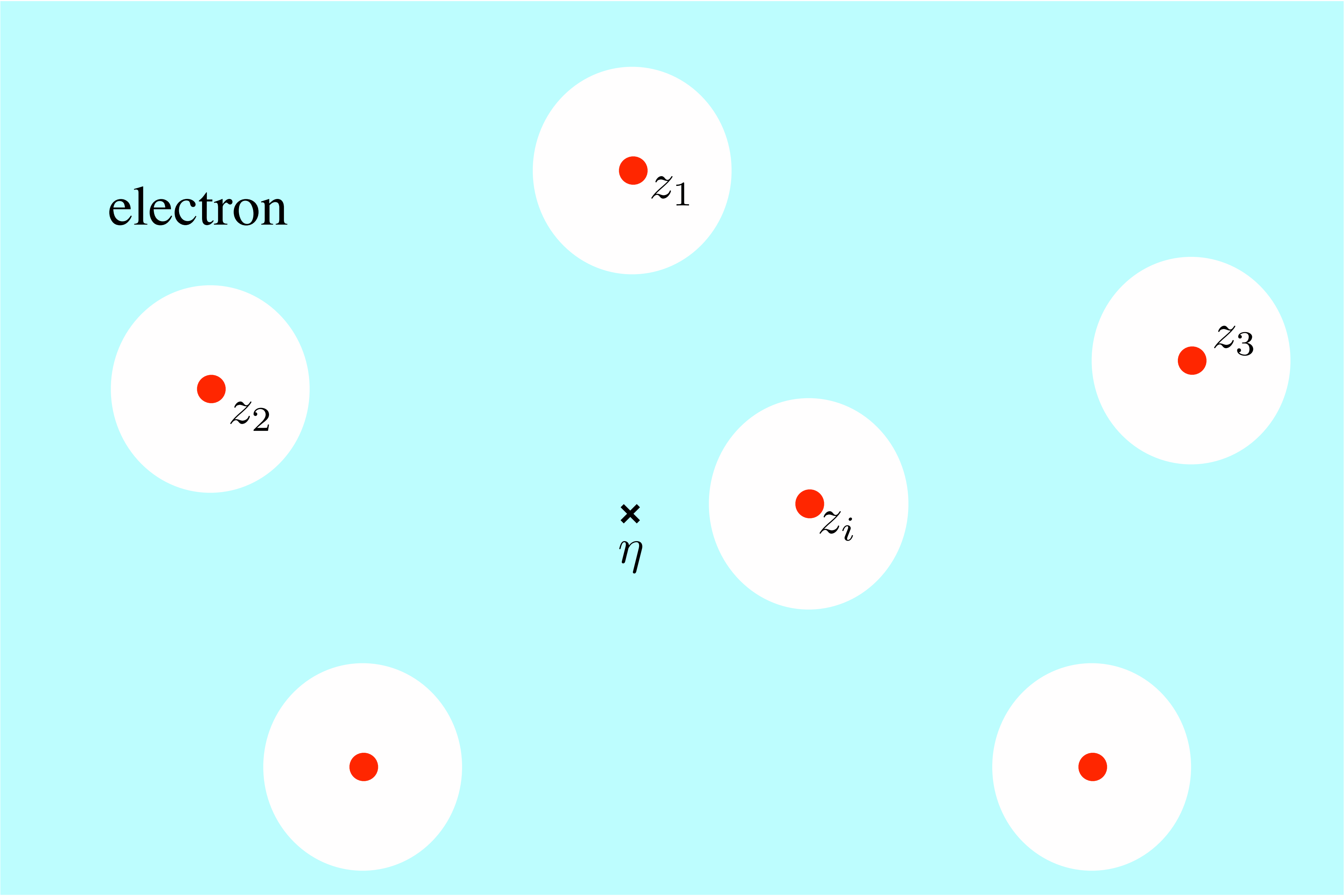

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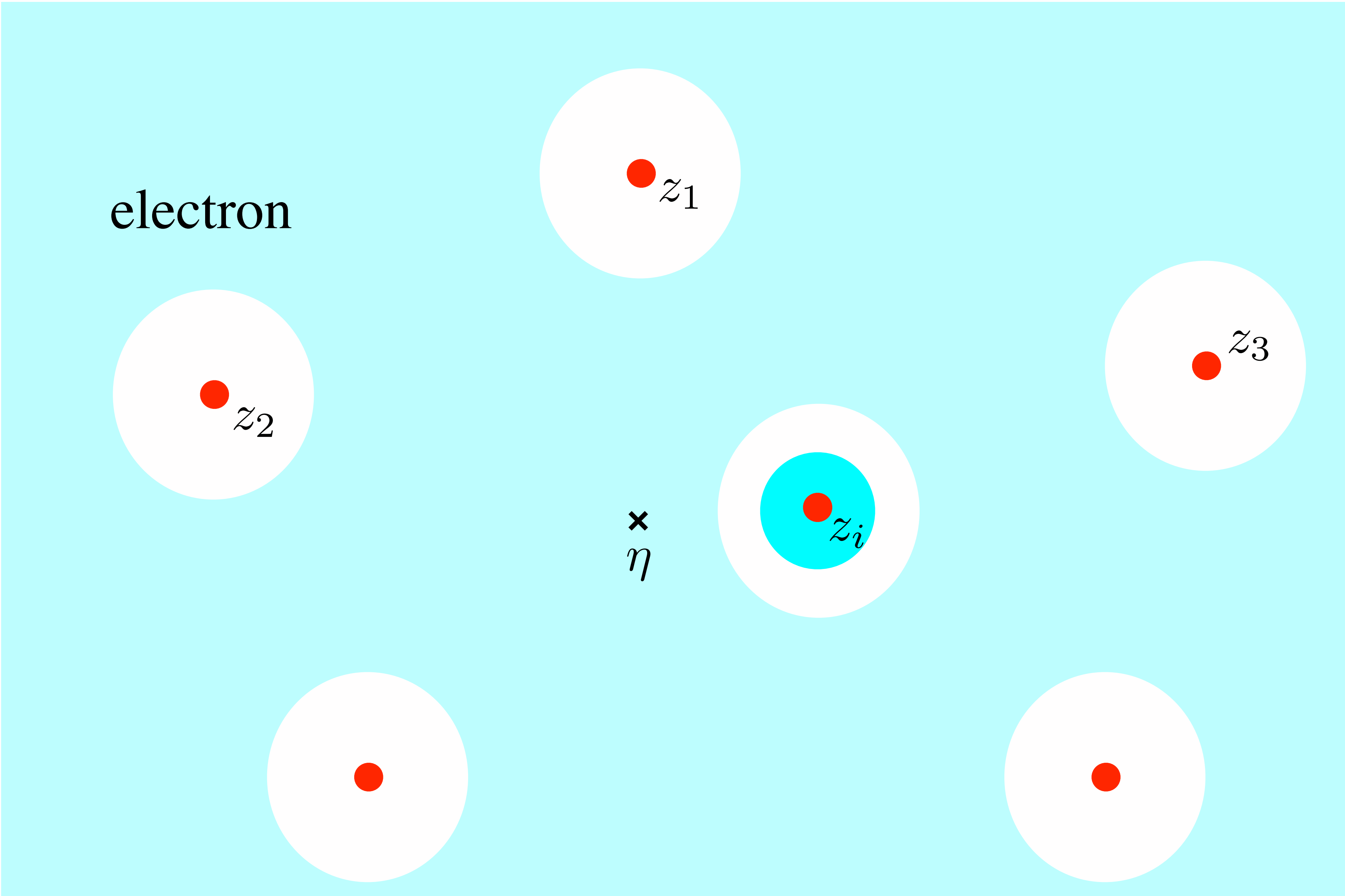

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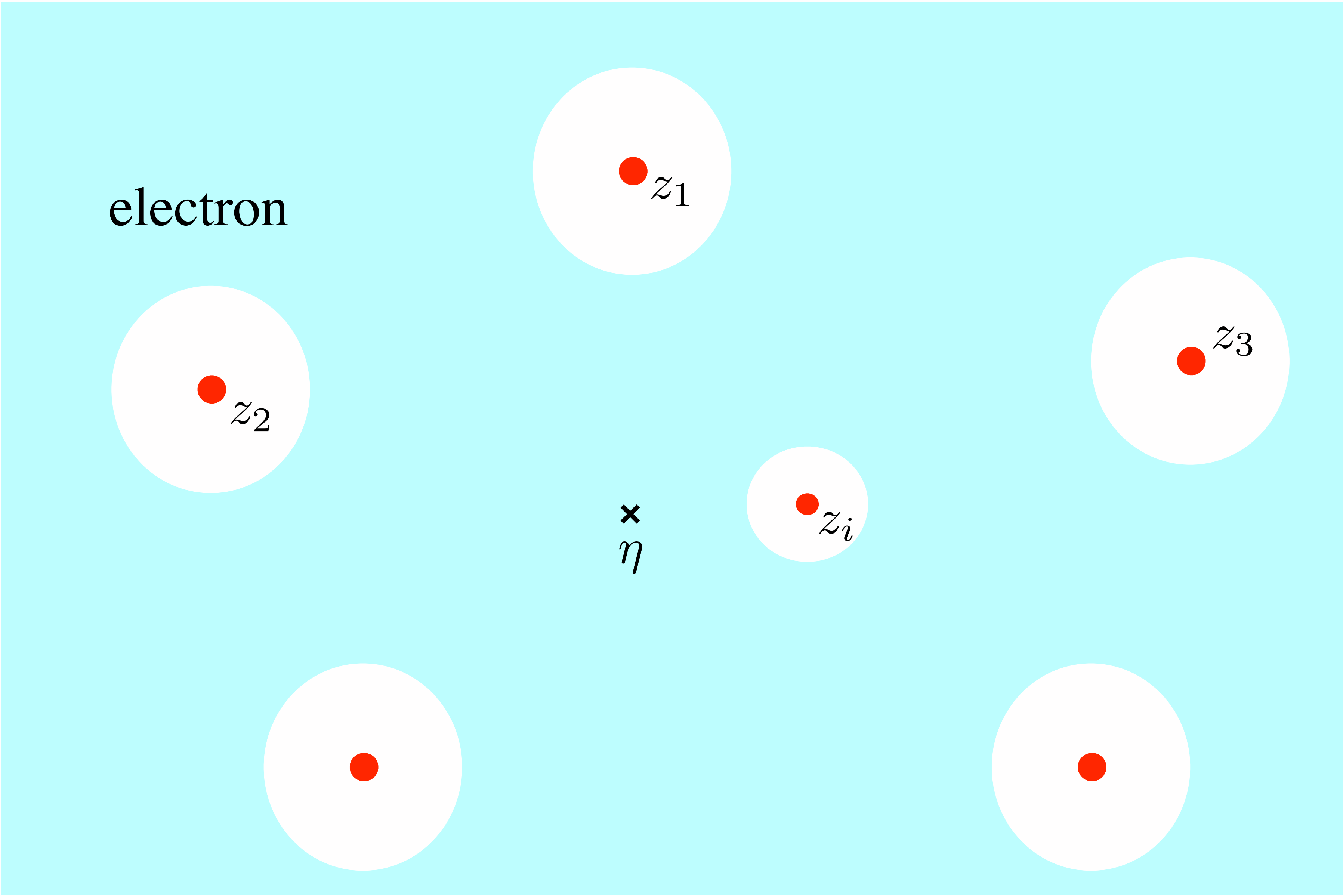

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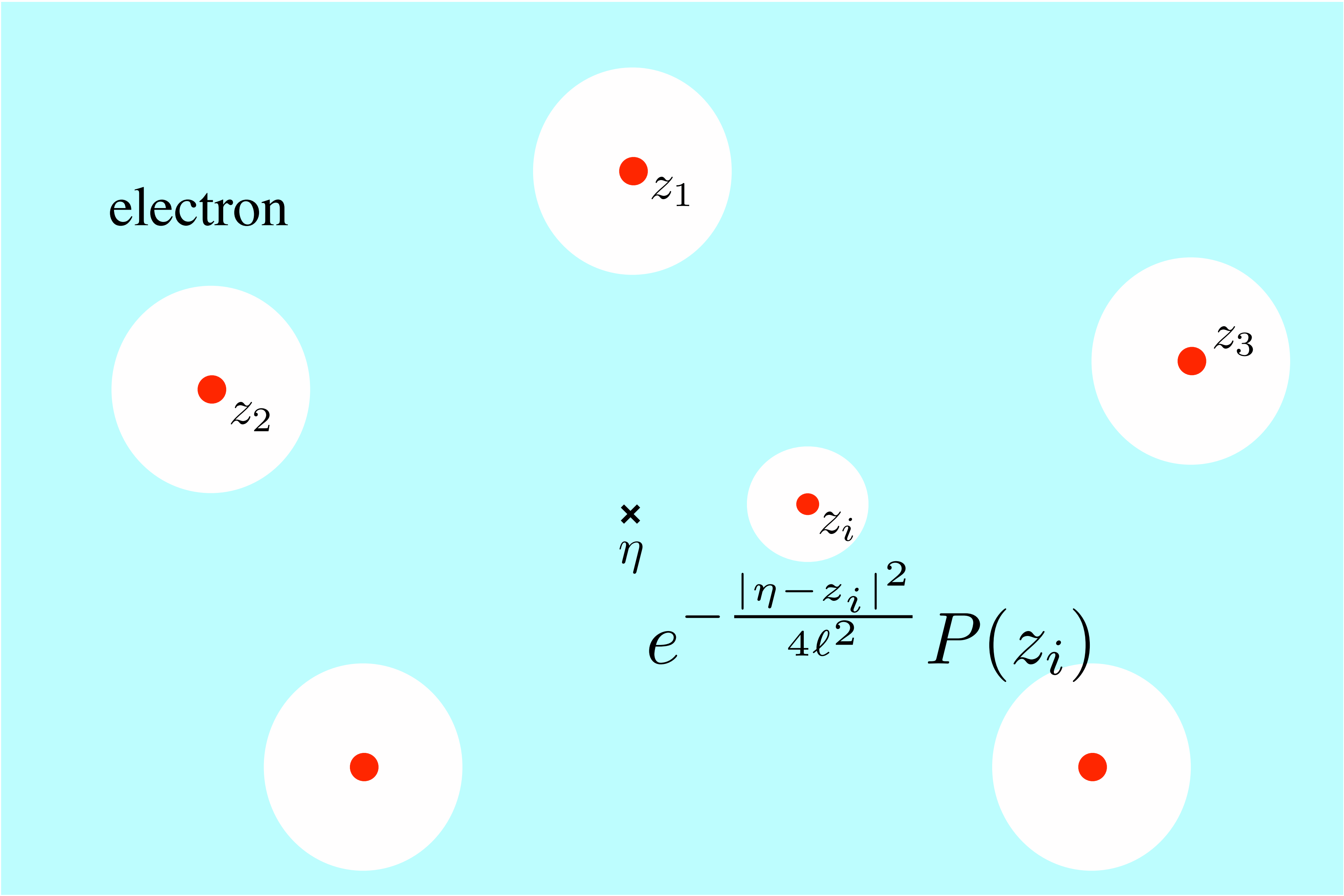


New quasielectron and quasihole operators

There exists operators $\mathcal{P}(\bar\eta)$, $\mathcal{H}(\eta)$ so that:

$$\Psi(\bar\eta_1 \ldots \bar\eta_{m_1}, \eta_1 \ldots \eta_{m_2}; z_1 \ldots z_N) = \langle \mathcal{H}(\eta_1) \ldots \mathcal{P}(\bar\eta_{m_2}) V_1(z_1) \ldots V_n(z_N) \rangle$$

- $\mathcal{P}(\bar\eta)$ / $\mathcal{H}(\eta)$ **are quasi-local on the magnetic length scale**
- $\mathcal{P}(\bar\eta)$ / $\mathcal{H}(\eta)$ **codes the charge and conformal spin of the quasiparticles**
- **Braiding phases can be calculated from monodromies, under the assumption that there are no additional Berry phases.**

These operators can be used at any level of the Haldane-Halperin hierarchy

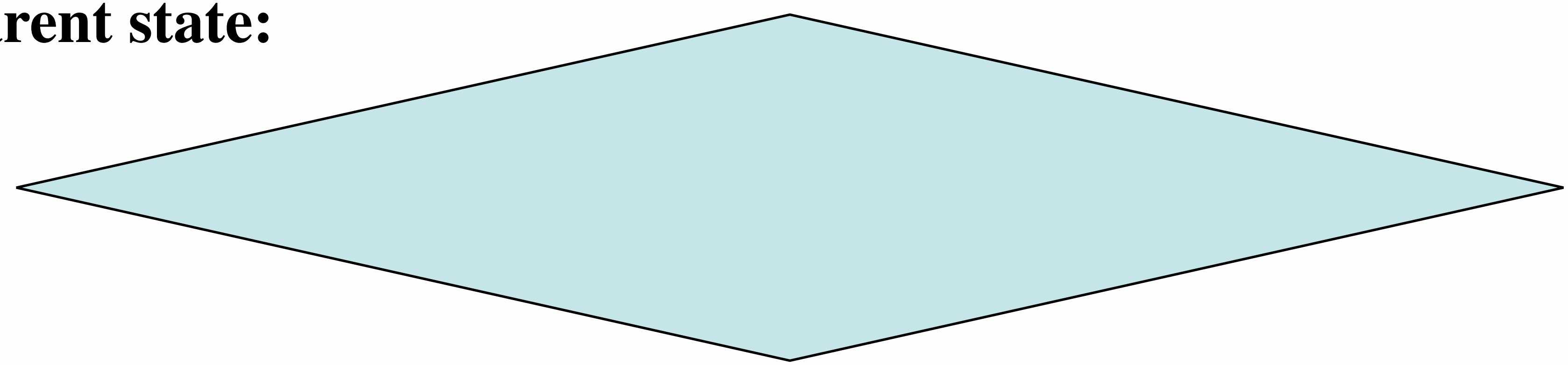
1^{st} application:

Hierarchy form of Jains wave functions

Quantum Hall quasielectron operators
in conformal field theory
T. H. Hansson, M. Hermanns, and S. Viefers
Phys. Rev. B **80**, 165330 (2009); arXiv:0903.0937.

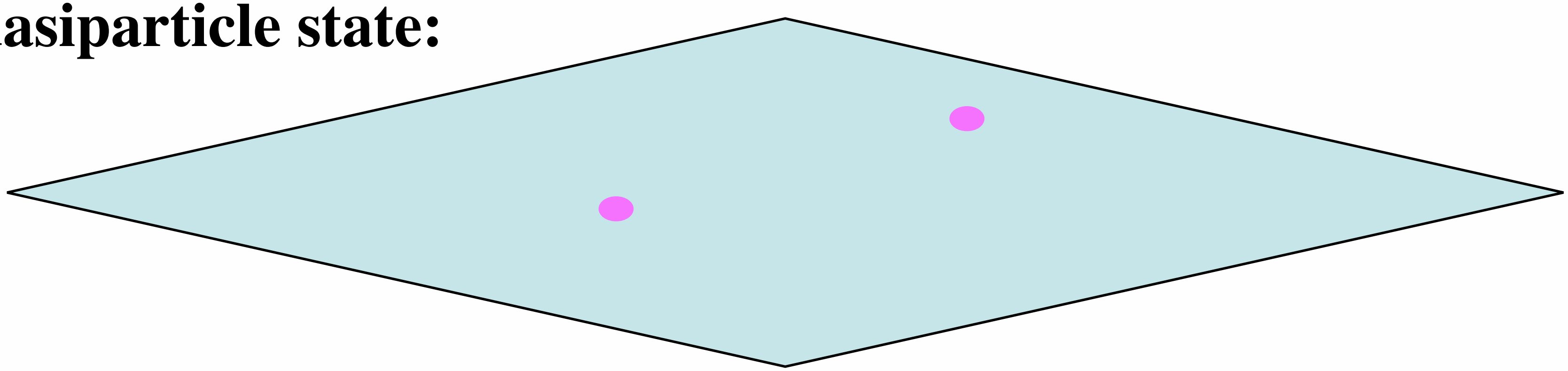
The Haldane-Halperin hierarchy idea

Parent state:



The Haldane-Halperin hierarchy idea

Quasiparticle state:



The Haldane-Halperin hierarchy idea

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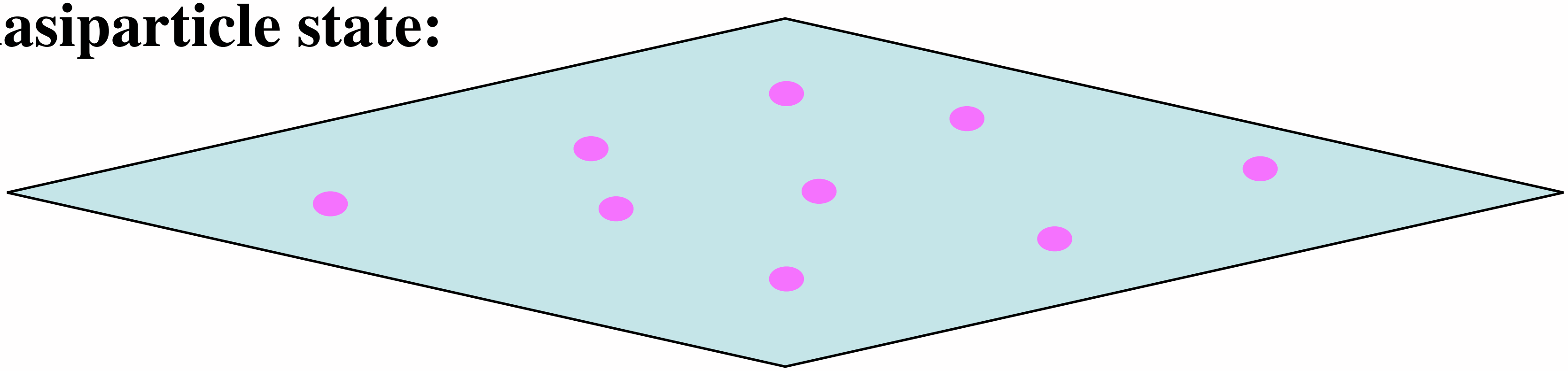
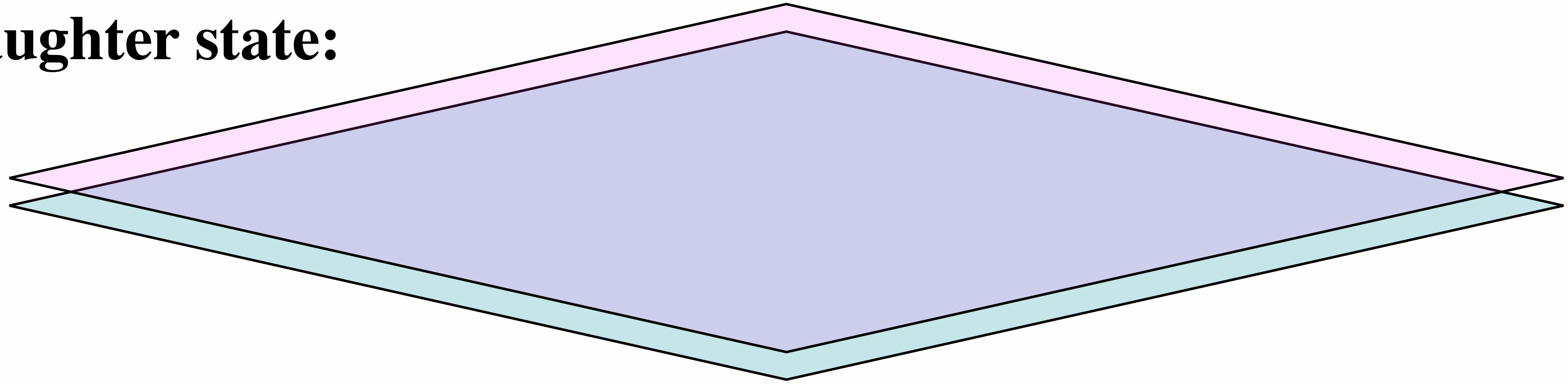


The Haldane-Halperin hierarchy idea

Daughter state:



The Haldane-Halperin hierarchy wave functions:

$$\Psi^{HH}(\{z_i\}) = \int [d^2\eta_k] \Psi^*_{\mathrm{qp}}(\vec{\eta}_1, \ldots, \vec{\eta}_M) \Psi(\vec{\eta}_1, \ldots, \vec{\eta}_M; z_1, \ldots, z_N)$$

"Pseudo wave function" for the quasiparticles

Multi quasiparticle electronic wave function

Complicated integral expressions!

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But the complication depends on the actual choice of the quasiparticle wave function $\Psi(\vec{\eta}_1 \ldots \vec{\eta}_M; z_1 \ldots z_N)$ **!**

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But the complication depends on the actual choice of the quasiparticle wave function $\Psi(\vec{\eta}_1 \ldots \vec{\eta}_M; z_1 \ldots z_N)$ **!**

The obvious, and original, idea was to use generalizations of,

$$\Psi_{\frac{1}{3},1qh}(\eta; z_1, \ldots z_N) = \prod_{i<j} (z_i - z_j)^3 \prod_i (z_i - \eta) e^{-\frac{1}{4}\sum_i |z_i|^2}$$

to generate $\Psi(\eta_1 \ldots \eta_M; z_1 \ldots z_N)$ **etc.**

But if we instead use:

$$\Psi(\bar{\eta}_1 \ldots \bar{\eta}_M; z_1 \ldots z_N) = \langle \mathcal{P}(\bar{\eta}_1) \ldots \mathcal{P}(\bar{\eta}_M) V(z_1) \ldots V(z_N) \rangle$$

the integrals over the quasiparticles coordinates can be done analytically to give explicit expressions for the hierarchy wave functions that turn out to be identical to the

The Composite Fermion wave functions in the positive Jain series:

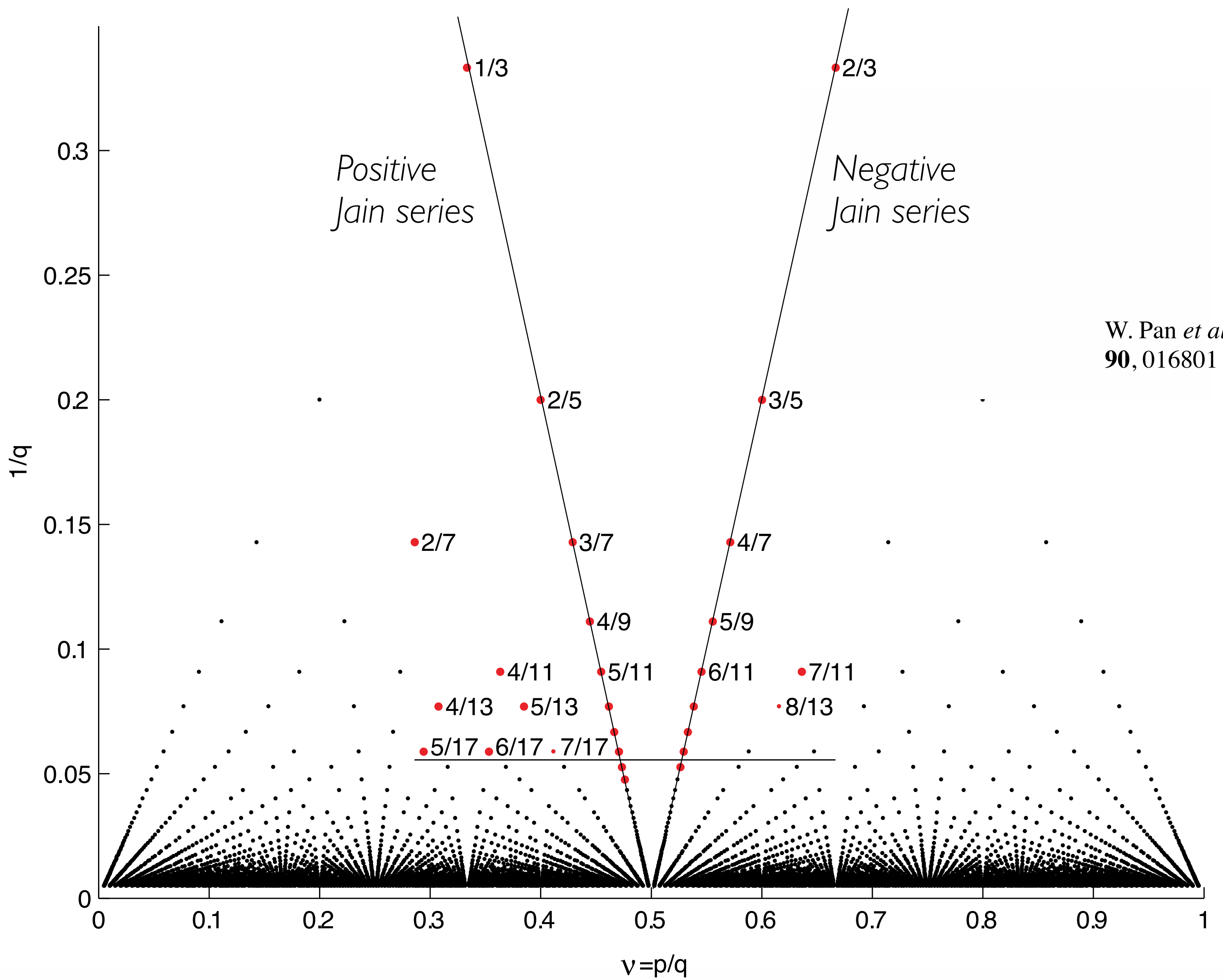
$$\Psi^{Jain}(\{z_i\}) = \mathcal{P}_{LLL} \left[\Psi_{nLL}(\{z_i, \bar{z}_i\}) \prod_{i<j} (z_i - z_j)^2 \right] e^{-\frac{1}{4\ell^2} \sum_i |z_i|^2}$$

Projector onto the LLL

n filled Landau levels of Composite Fermions

Vortex or “flux” attachment factor

The Abelian Quantum Hall Hierarchy



W. Pan *et al*., Phys. Rev. Lett. **90**, 016801 (2003).

Comments:

- **Generalizes to negative Jain series**
- **Generalizes to the full hierarchy**
- **A general hierarchy ground state can be written:**

$$\Psi = \mathcal{A}\langle \prod_{\alpha=1}^{n} \prod_{i_\alpha \in I_\alpha} V_\alpha(z_{i_\alpha}) \rangle_{bg}$$

where

$$V_\alpha(z) =: \partial_z^{\alpha-1} e^{i\sum_\beta Q_{\alpha\beta}\varphi^\beta(z)} : \qquad \alpha = 1, 2 \ldots n$$

where the matrix Q is related to the K-matrix by: $\mathbf{K} = \mathbf{Q}\mathbf{Q}^T$

and the spin vector is given by: $s_\alpha = \frac{1}{2} K_{\alpha\alpha} + \alpha - 1$

***i.e. n* distinct electron operators at the n$^{\text{th}}$ hierarchy level.**

Note the presence of derivatives, which appear when "fusing" the electron operator with the inverse hole using OPE !!

2^{nd} application:

Hierarchy wave functions on the Torus

Hall viscosity of Hierarchical Quantum Hall States
M. Fremling, T.H. Hansson and J. Suorsa
Phys. Rev. B **89**, 125303 (2014).

The Laughlin wave function on a torus

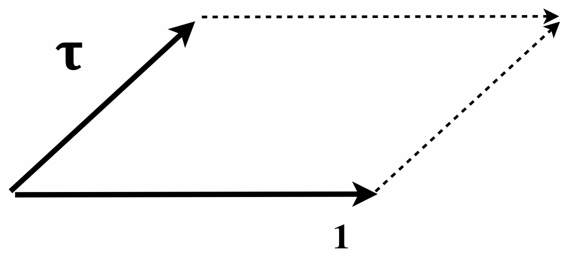
$$\psi_s = \mathcal{N}_0\left[\sqrt{\tau_2}\,\eta(\tau)^2\right]^{qN_e/2}\prod_{i<j}^{N_e}\left[\frac{\vartheta_1\left(\frac{z_i-z_j}{L}|\tau\right)}{\eta(\tau)}\right]^q \times \mathcal{F}_s(Z)e^{i\pi\tau N_\Phi \sum_{i=1}^{N_e} y_i^2}$$

where τ is the modular parameter of the torus, can be extracted from,

$$\langle V(z_1,\bar z_1)\cdots V(z_N,\bar z_N)\mathcal{O}_{bg}\rangle = \sum_{e,m\in\mathbb{Z}} \Psi_{e,m}\bar\Psi_{e,-m}$$

by imposing periodic boundary conditions:

$$T_a|\psi\rangle = e^{i\phi_a}|\psi\rangle \quad ; \quad a=1,2$$



Where T_a are magnetic translations along the periods of the torus.

This construction directly generalized to multi-component states, describing e.g. multilayers. But for the hierarchy states the wave functions involve derivatives, which do not respect the boundary conditions

Define

$$t_{m,n} = e^{i\frac{L}{N_\Phi}(nR^1 - mR^2)} \quad ; \quad m,n = 1,2,\ldots N_\Phi$$

so $$T_1 = t_{1,0}^{N_\Phi} \quad \text{and} \quad T_2 = t_{0,1}^{N_\Phi}$$

Then a general "derivative" that preserves the boundary conditions is

$$\mathbb{D}_{(\alpha)} \text{ "=" } \sum_{m,n=0}^{2qN_\Phi} \lambda_{m,n}^{N_\alpha} T_{m,n}^{(\alpha)} \quad \text{with} \quad T_{m,n}^{(\alpha)} = \prod_{i_\alpha \in I_\alpha} t_{m,n}^{(i_\alpha)}$$

where $\lambda_{m,n}^{N_\alpha}$ are complex coefficients to be determined.

Modular properties

Laughlin (schematically)

$$\psi_s \xrightarrow{\mathcal{S}} \left(\frac{\tau}{|\tau|}\right)^{h_{tot}} U_{\mathcal{S}} \sum_{s'=1}^{m} \mathcal{S}_{s,s'}\psi_{s'}$$

$$\psi_s \xrightarrow{\mathcal{T}} U_{\mathcal{T}} \sum_{s'=1}^{m} \mathcal{T}_{s,s'}\psi_{s'}$$

$$\mathcal{T}: \quad \tau \to \tau + 1$$

$$\mathcal{S}: \quad \tau \to -\frac{1}{\tau}$$

With $\lambda_{m,n} = \sqrt{\tau_2}\eta^3(\tau)\frac{e^{-i\pi\tau n^2\epsilon^2}e^{-i\pi nm\epsilon^2}}{\vartheta_1(m\epsilon + n\epsilon\tau|\tau)}$ $\qquad \epsilon = 1/N_\Phi$

$$\boxed{\begin{aligned} \mathcal{S} &: \quad \mathbb{D}_{(\alpha)} \to \left(\frac{\tau}{|\tau|}\right)^{N_\alpha} U_{\mathcal{S}}\,\mathbb{D}_{(\alpha)}U_{\mathcal{S}}^\dagger \\ \mathcal{T} &: \quad \mathbb{D}_{(\alpha)} \to U_{\mathcal{T}}\,\mathbb{D}_{(\alpha)}U_{\mathcal{T}}^\dagger\,. \end{aligned}}$$

$$\boxed{[\mathbb{D}_{(\alpha)}, \mathbb{D}_{(\beta)}] = 0}$$

$$\mathbb{D}_{(\alpha)} \to \prod_{j\in I_\alpha} \partial_{z_j}$$

Hierarchy wave functions on the torus

Recall, at the n[th] level of the hierarchy, we have n operators,

$$V_\alpha(z) =: \underline{\partial_z^{\alpha-1}} e^{i\sum_\beta Q_{\alpha\beta}\varphi^\beta(z)} : \qquad \alpha = 1, 2 \ldots n$$

where the matrix Q is related to the K-matrix by: $\mathbf{K} = \mathbf{Q}\mathbf{Q}^T$

and the spin vector is given by: $s_\alpha = \frac{1}{2}K_{\alpha\alpha} + \alpha - 1$

A general hierarchy wave function on the plane is now given by

$$\Psi = \mathcal{A}\langle \prod_{\alpha=1}^{n} \prod_{i_\alpha \in I_\alpha} V_\alpha(z_{i_\alpha})\mathcal{O}_{bg}\rangle$$

The torus wave function is obtained by the substitution: $\prod_{j\in I_\alpha} \partial_{z_j}^{\alpha-1} \to \mathbb{D}_{(\alpha)}^{\alpha-1}$

Result for the $\nu = 2/5$ Jain wave function:

The CFT wave function is obtained by calculating the pertinent conformal blocks and then acting with the operator $\mathbb{D}$.

The figures show the overlaps with the numerically generated Coulomb wave functions.
No variational parameter!

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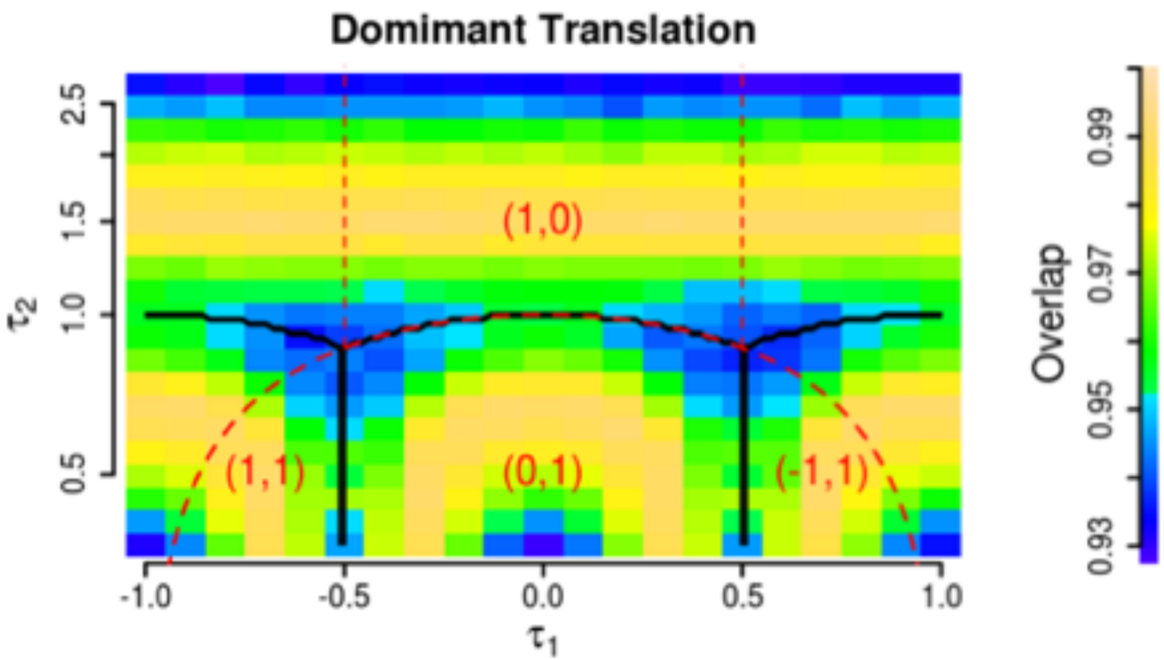

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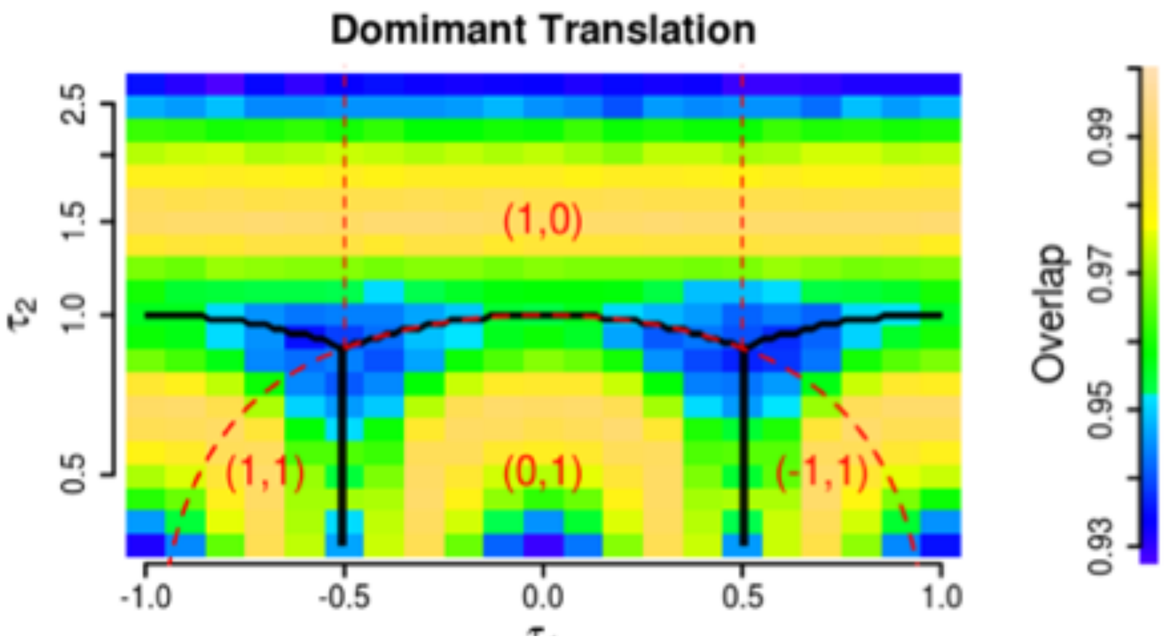

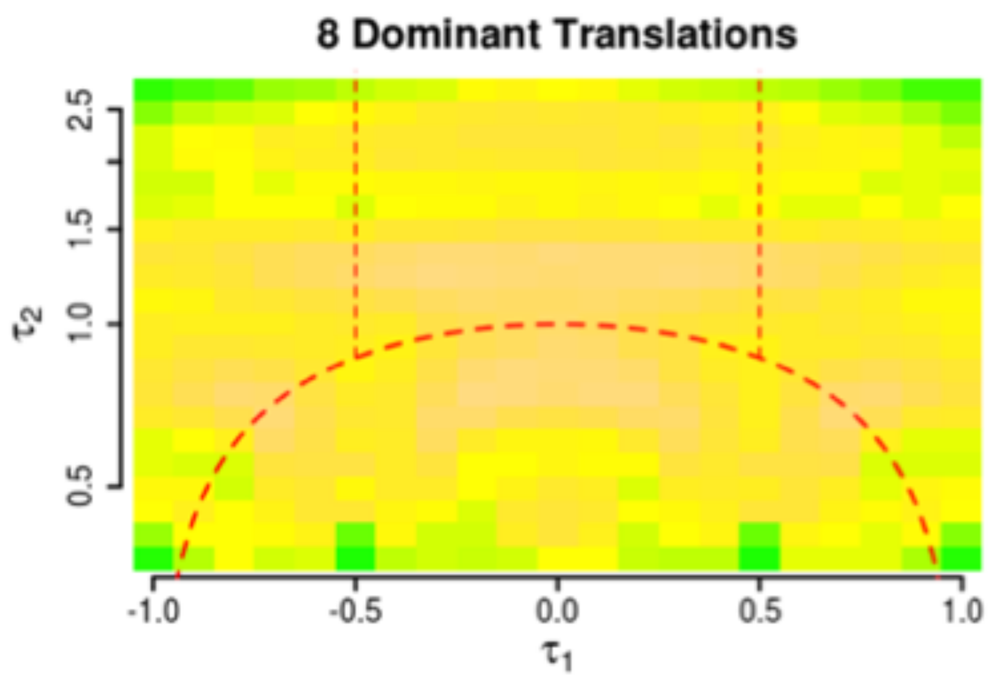

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So where did the idea enter?

On the plane:

$$V_1(z)H^\star(w) \to N[V_1(z)H^\star(w)]$$

by OPE $= \partial_z e^{i(\sqrt{q}-1/\sqrt{q})\varphi(z)} \equiv \partial_z \tilde{V}_2$

On the torus:

$$N[V_1(z)H^\star(w)] = e^{K(\delta)} t_\delta e^{i(\sqrt{q}-1/\sqrt{q})\varphi(z)} \equiv e^{K(\delta)} t_\delta \tilde{V}_2(z)$$

keeping a *finite* distance: $\delta = w - z$

where $K(z,\bar{z}) = -\ln\left|\frac{L\vartheta_1(z/L|\tau)}{\vartheta_1'(0|\tau)} e^{i\pi\tau y^2}\right|^2$ is the torus two-point function

gives the $\lambda_{m,n}$ with the correct modular properties.

3nd application:

Matrix Product State representation of hierarchy wave functions

E. Ardonne, J. Dubail, J. Kjäll T. H. H.,
work in progress

Structure of the general LLL wave function:

$$\Psi(z_1 \dots z_N) = \int [d^2\xi_i] \, \langle z_1 \dots z_N | \xi_1 \dots \xi_N \rangle \, \Psi_{K,\kappa,\mathbf{S},\mathbf{s}}(\xi_1, \dots \xi_N, \bar{\xi}_1, \dots \bar{\xi}_N)$$

Coherent state kernel

Guiding center coordinates

$$\Psi_{K,\kappa,\mathbf{S},\mathbf{s}}(\{\xi_i\}, \{\bar{\xi}_i\}) = \mathcal{A} \prod_\alpha \partial_{\xi_\alpha}^{s_\alpha} \partial_{\bar{\xi}_\alpha}^{\bar{s}_\alpha} \prod_{\alpha<\beta} (\xi_\alpha - \xi_\beta)^{\kappa_{\alpha\beta}} (\bar{\xi}_\alpha - \bar{\xi}_\beta)^{\bar{\kappa}_{\alpha\beta}}$$

Topological data

- **The CFT "representative wave function" is interpreted as the wave function in the basis of coherent states.**
- **The coherent state kernel depends on details, such as variations in the magnetic field, and anisotropies due to the lattice.**
- **The description easily generalizes to higher Landau level, by choosing a pertinent non-holomorphic kernel.**

4^{nd} application:

How to condense nonabelian anyons

Condensing non-Abelian quasiparticles
M. Hermanns
Phys. Rev. Lett. **104**, 056803 (2010); arXiv: 0906.2073

Moore-Read quasielectrons:

$$\mathcal{P}(\bar\eta) = \int d^2w\, e^{\frac{1}{2m}\bar\eta w} \left(\tilde H^{-1}\, \bar\partial_w J_p(w) \right)^\star$$

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Generalized normal ordering

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Generalized normal ordering

Ising representation

$$V(z) = \psi(z) e^{i\sqrt{2}\varphi(z)}$$

$$H^{-1}(z) = \sigma(z) e^{-\frac{i}{2\sqrt{2}}\varphi(z)}$$

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Does not (directly) give holomorphic wf:s...

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Bosonic representation

$$V(z) = \cos(\phi(z)) e^{i\sqrt{2}\varphi(z)}$$

$$H_{\pm}(\eta) = e^{\pm i\phi(\eta)/2} e^{\frac{i}{2\sqrt{2}}\varphi(z)}$$

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$$H_\pm(\eta) = e^{\pm i\phi(\eta)/2} e^{\frac{i}{2\sqrt{2}}\varphi(z)}$$

$$H^{(b)}_\pm(\eta) = e^{\frac{i}{\sqrt{8}}\varphi} e^{\pm\frac{i}{2}\phi} e^{i\sqrt{\frac{3}{8}}\phi_1 \pm \frac{i}{2}\varphi_2}$$

Moore-Read quasielectrons:

$$\mathcal{P}(\bar\eta) = \int d^2w\, e^{\frac{1}{2m}\bar\eta w} \left(\tilde H^{-1}\, \bar\partial_w J_p(w)\right)^\star$$

Generalized normal ordering

Ising representation

$$V(z) = \psi(z) e^{i\sqrt{2}\varphi(z)}$$

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Which (unfortunately) comes out like:

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$$\begin{aligned}
\Psi_{(\eta_1,\eta_2)(\eta_3,\eta_4)} &= \sum_I (-1)^{\sigma(I)} e^{(\bar\eta_1 z_\alpha + \bar\eta_2 z_\beta + \bar\eta_3 z_\gamma + \bar\eta_4 z_\delta)/8} \partial_\alpha \partial_\beta \partial_\gamma \partial_\delta \\
&\times \left[\psi_{(\alpha,\beta)(\gamma,\delta)} (z_\alpha - z_\beta)(z_\gamma - z_\delta) \prod_{i<j \notin I} (z_i - z_j)^2 \prod_{\substack{a \in I \\ j \notin I}} (z_a - z_j) \right] \\
\psi_{(\alpha,\beta)(\gamma,\delta)} &= \left(\frac{(z_\alpha - z_i)(z_\beta - z_i)(z_\gamma - z_j)(z_\delta - z_j) + (i \leftrightarrow j)}{z_i - z_j} \right)
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These functions span the expected 2-dim Hilbert space for4 nonabelian quasielectrons.
The nonabelian statistical matrix is however coded in the Berry matrix rather than in the monodromies.

Hierarchical NA-states?

A Genuine Non-Abelian Hierarchy state

Following the work of Maria Hermanns, we consider the correlator,

$$\Psi_\xi = \langle \mathcal{P}_{\xi(1)}(\eta_1)\mathcal{P}_{\xi(2)}(\eta_2)\ldots\mathcal{P}_{\xi(n)}(\eta_n)\prod_{i=1}^{N} V(z_j)\rangle$$

where ξ is a string of equally many + and - , multiply with an appropriate pseudo wave function and integrate over η_i to get holomorphic hierarchy states.

Hierarchical NA-states?

A simple way to generate non-Abelian hierarchy states is to just multiply a non-Abelian state, say the Moore-Read Pfaffian, with a symmetric Abelian hierarchy state. This amounts to condensing abelian quasiparticles. (Bonderson - Slingerland hierarchy)

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What is the character of such a state state?

From $SU(2)_2$ to $SU(3)_2$:

A non-Abelian condensate in the bosonic MR state

From SU(2)$_2$ to SU(3)$_2$:

A non-Abelian condensate in the bosonic MR state

$$\psi_{4/3} = \mathcal{S}\left[\prod_{j\in I_1}\partial_j(1-1)^2(2-2)^2(1-2)\times\prod_{j\in I_3}\partial_j(3-3)^2(4-4)^2(3-4)\right],$$

$$(i-i) = \prod_{\alpha<\beta\in I_i}(z_\alpha - z_\beta) \qquad (i-j) = \prod_{\alpha\in I_i,\beta\in I_j}(z_\alpha - z_\beta)$$

Close relative to the NASS state

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Close relative to the NASS state

$$\Psi_{4/3} \quad = \mathcal{S}\left[\langle \textstyle\prod_{j=1}^{N/2}\partial_j V_+(z_j)\prod_{j=N/2+1}^{N} V_-(z_j)\rangle\right]$$

$$V_+ = \psi_1(z) e^{i\sqrt{\frac{3}{4}}\varphi_c(z) + \frac{i}{2}\varphi_s(z)}$$

$$V_- = \psi_2(z) e^{i\sqrt{\frac{3}{4}}\varphi_c(z) - \frac{i}{2}\varphi_s(z)}$$

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ψ_1 and ψ_2 are $su(3)_2/[u(1)]^2$ parafermions

Thank you for listening!

More details in these references:

Conformal Field Theory of Composite Fermions
T.H. Hansson, C.-C. Chang, J.K. Jain and S. Viefers.
Phys. Rev. Lett. **98**, 076801 (2007); arXiv:cond-mat/0603125.

Composite-fermion wave functions as correlators in conformal field theory
T. H. Hansson, C.-C. Chang, J. K. Jain, and S. Viefers
Phys. Rev. B **76**, 075347 (2007); arXiv:0704.0570.

Microscopic theory of the quantum Hall hierarchy
E.J. Bergholtz, T. H. Hansson, M. Hermanns, and A. Karlhede
Phys. Rev. Lett. **99**, 256803 (2007); arXiv:cond-mat/0702516.

Quantum Hall wave functions on the torus
M. Hermanns, J. Suorsa, E.J. Bergholtz, T.H. Hansson, and A. Karlhede
Phys. Rev. B **77**, 125321 (2008), arXiv:0711.4684.

Hierarchy wave functions–from conformal correlators to Tao-Thouless states
E.J. Bergholtz, T. H. Hansson, M. Hermanns, A. Karlhede, and S. Viefers
Phys. Rev. B **77**, 165325 (2008); arXiv:0712.3848.

Quantum Hall Quasielectrons - Abelian and Non-Abelian
T. H. Hansson, M. Hermanns, N. Regnault, and S. Viefers
Phys. Rev. Lett. **102**, 166805 (2009), arXiv:0810.0636.

Quantum Hall quasielectron operators in conformal field theory
T. H. Hansson, M. Hermanns, and S. Viefers
Phys. Rev. B **80**, 165330 (2009); arXiv:0903.0937.

Condensing non-Abelian quasiparticles
M. Hermanns
Phys. Rev. Lett. **104**, 056803 (2010); arXiv:0906.2073

Quasihole condensates in quantum Hall liquids
J. Suorsa, S. Viefers, and T.H. Hansson
Phys. Rev. B **83**, 235130 (2011); arXiv:1004.3657.

A general approach to quantum Hall hierarchies
J. Suorsa, S. Viefers, and T.H. Hansson
New Journ. of Phys. **13**, 075006 (2011); arXiv:1011.5365.

The quantum Hall hierarchy in spherical geometry
T. Kvorning
Phys. Rev. B **80**, 165330 (2009); arXiv:0903.0937.

Hall viscosity of Hierarchical Quantum Hall States
M. Fremiling, T.H. Hansson and J. Suorsa
Phys. Rev. B **89**, 125303 (2014); arXiv:1312.6038.

The quasielectron operator - details

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$$\Psi_{1qe}^{(m)}(\vec{R}\,; z_1 \ldots z_N) = \langle \mathcal{P}(\vec{R}) V_e(z_1) \ldots\ldots V_e(z_N) \rangle$$

$$\begin{aligned} \mathcal{P}(\vec{R}) &= e^{-\frac{|\eta|^2}{4m}} \mathcal{P}(\bar{\eta}) \quad ; \quad \eta = X + iY \\ &= e^{-\frac{|\eta|^2}{4m}} \int d^2w \, e^{\frac{1}{2m}\bar{\eta} w} \left(\tilde{H}^{-1}(z) \, \bar{\partial}_w J_p(w) \right) \end{aligned}$$

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- $\mathcal{P}(\bar{\eta})$ **is quasi-local on the magnetic length scale**
- $\mathcal{P}(\bar{\eta})$ **has same charge and same conformal dim. as** $H^{-1}(z)$
- **Multiple insertions of** $\mathcal{P}(\bar{\eta})$ **gives multi-quasielectron states**

With this we:

- ✓ Get explicit wave functions for all ground states and quasiparticle excitations in the plane
- ✓ Find the Laughlin and Jain states as special cases
- ✓ Generalize to the sphere and gives explicit expressions for the shift S. (T. Kvorning, Phys. Rev. B 87, 195131, 2013)
- ✓ Make the topological properties explicit in terms of K and S
- ✓ Provide a (minimal) edge theory for all states
- ✓ Get agreement with known result in the ThinTorus limit
- ✓ Allow for generalization to non-abelian hierarchies
- ✓ Construct torus wave functions with good modular properties, that yield a QH viscosity consistent with the shift on the sphere
- Get a scheme for improving the wave functions, while preserving the good topological properties