Nonabelian hierarchies

Maria Hermanns, UzK

collaborators:
Yoran Tournois, UzK

Hans Hansson, SU
Steve H. Simon, Oxford
Susanne Viefers, UiO
Quantum Hall hierarchies, arXiv:1601.01697

CRC183
ENTANGLED STATES OF MATTER

Emmy Noether-Programm
DFG Deutsche Forschungsgemeinschaft

Outline

Haldane-Halperin hierarchy

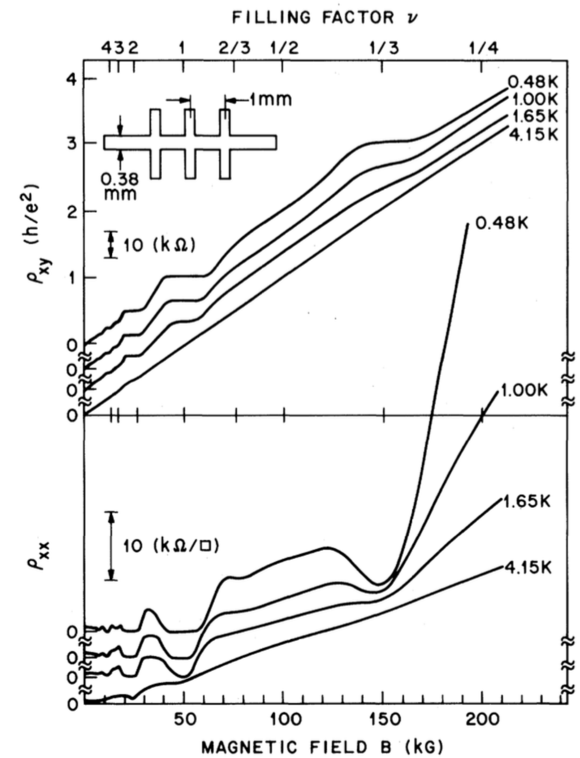
conformal field theory description of FQH states

nonabelian hierarchies

- Levin-Halperin
- Bonderson-Slingerland
- Hermanns

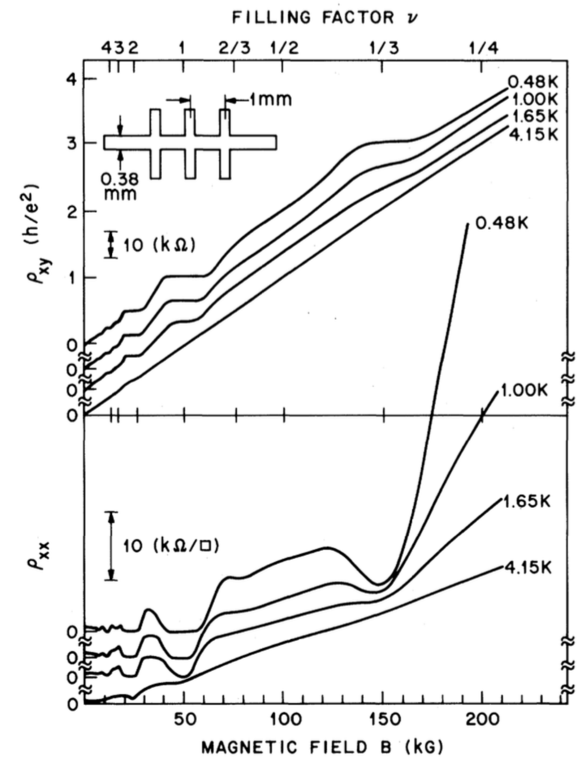
Laughlin's 1/3 wave function

$$\nu = 1 : \; \psi_1 = \prod_{i<j} (z_i - z_j)\, e^{-\sum_j |z_j|^2/(4\ell^2)}$$



Laughlin’s 1/3 wave function

$$\psi_{\frac{1}{3}} = \prod_{i<j} (z_i - z_j)^3 \, e^{-\sum_j |z_j|^2/(4\ell^2)}$$

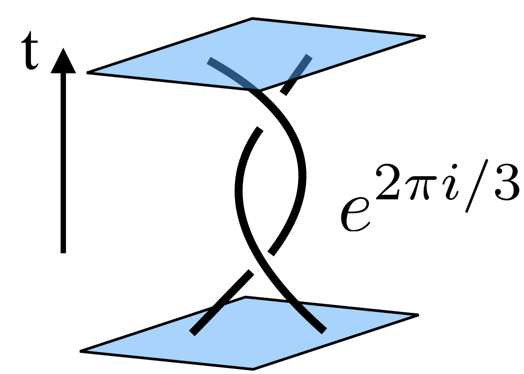


Laughlin ‘confirmed validity’ of wave function by overlap calculation with N=4 exact diagonalization ground state

<u>properties</u> (plasma analogy)

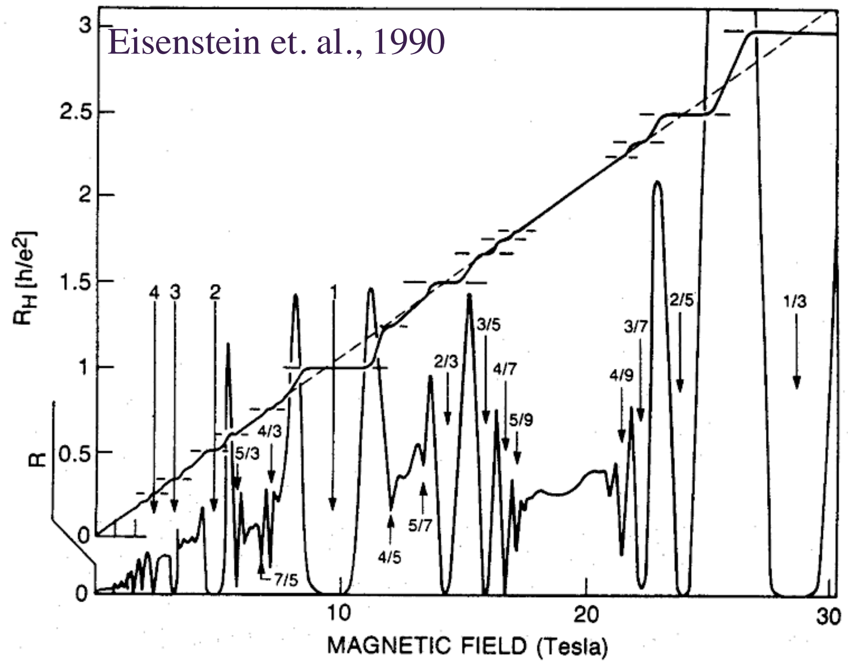
- $|\psi|^2 \sim$ partition function of a classical plasma
- screening
- uniform charge density
- incompressible liquid
- excitations have fractional charge e/3

- fractional statistics (Arovas, Schrieffer, Wilczek)



Fractional Quantum Hall effect

zoo of FQH states

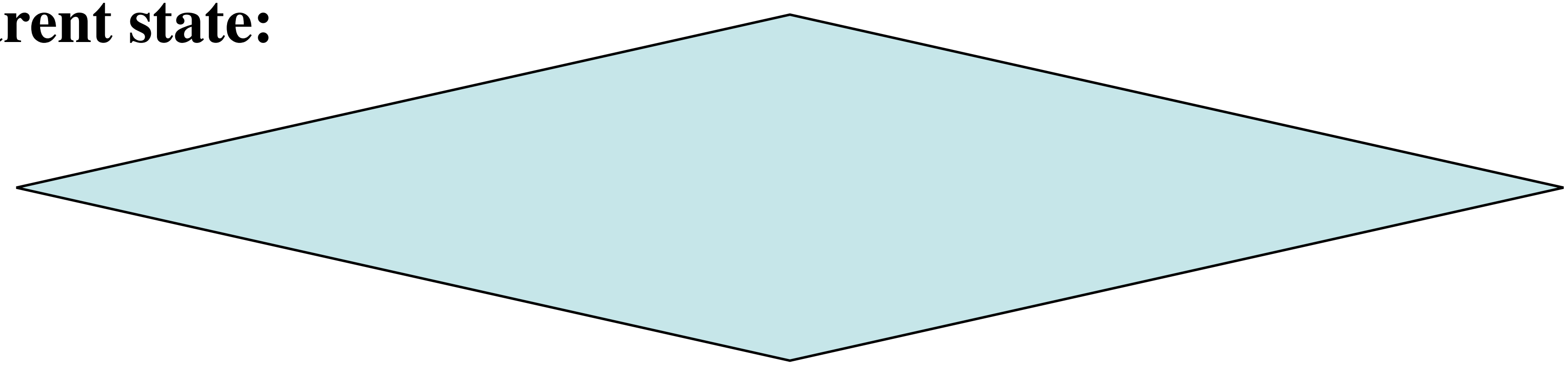


experimental signatures:

- fractional Hall conductance
- fractional electric charge
- (fractional statistics) controversial!

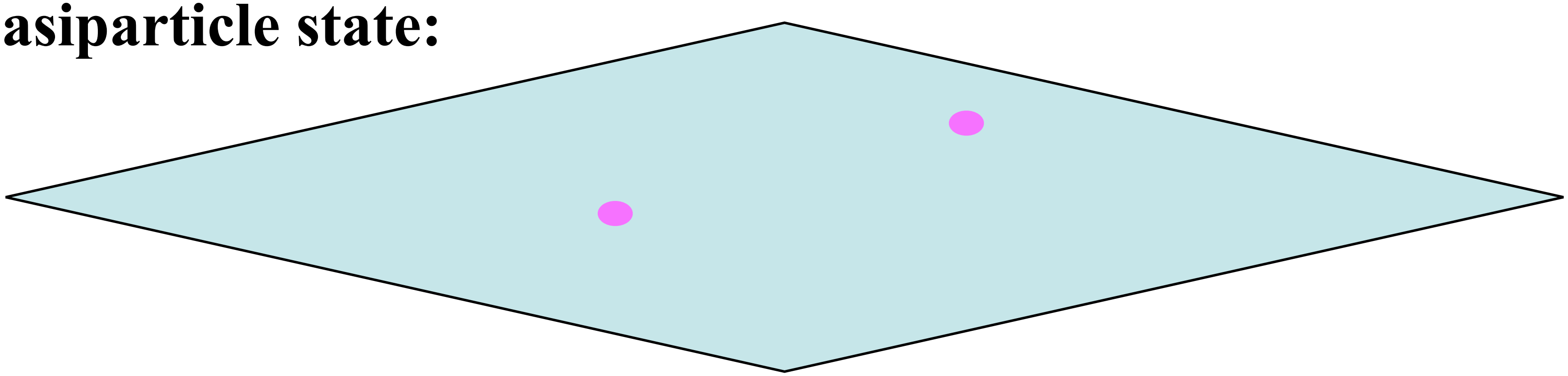
The Haldane-Halperin idea

Parent state:



The Haldane-Halperin idea

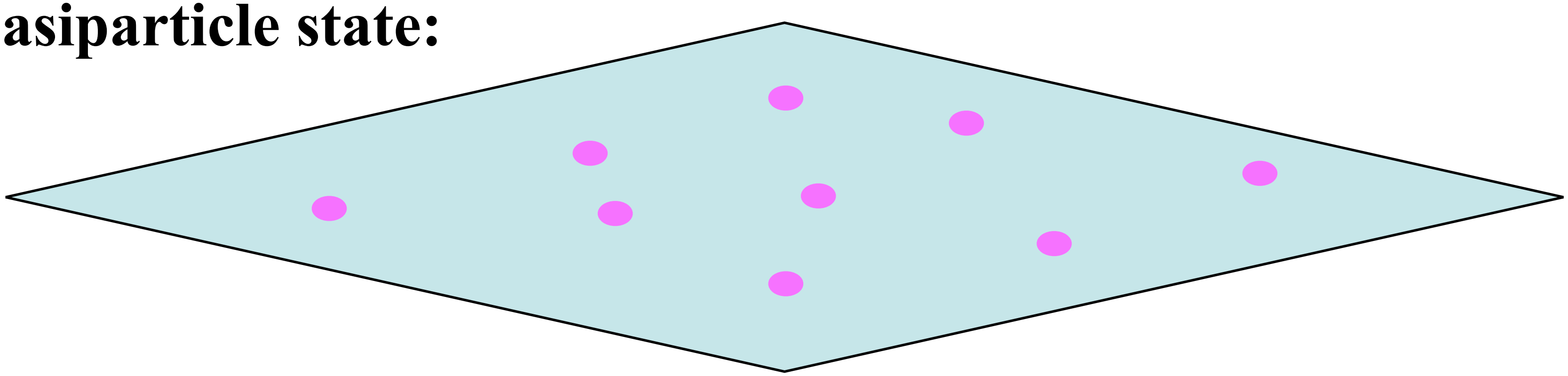
Quasiparticle state:



@Hans Hansson

The Haldane-Halperin idea

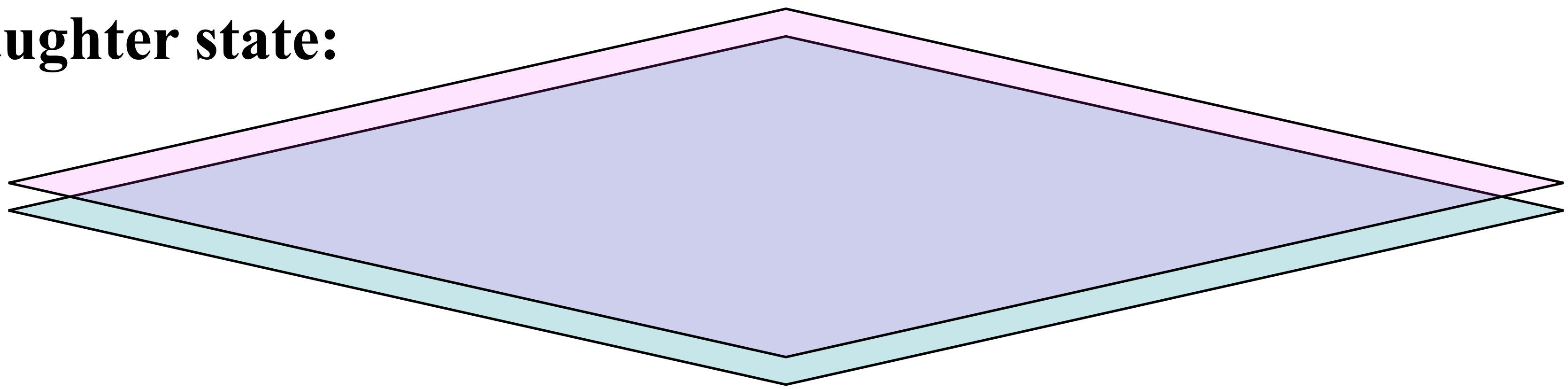
Quasiparticle state:



@Hans Hansson

The Haldane-Halperin idea

Daughter state:



@Hans Hansson

The Haldane-Halperin hierarchy

Haldane 1983, Halperin 1983

Halperin's proposal:

$$\Psi_{n+1}(z_1 \ldots z_N) = \int d^2\vec{R}_1 \ldots \int d^2\vec{R}_M \, \Phi^\star(\vec{R}_1 \ldots \vec{R}_M) \Psi_n(\vec{R}_1 \ldots \vec{R}_M; z_1 \ldots z_N)$$

ground state wave function at hierarchy level n+1

pseudo wave function

multi-quasiparticle wave function at hierarchy level n

The Haldane-Halperin hierarchy

Haldane 1983, Halperin 1983

Halperin's proposal:

$$\Psi_{n+1}(z_1 \dots z_N) = \int d^2\vec{R}_1 \dots \int d^2\vec{R}_M \, \Phi^\star(\vec{R}_1 \dots \vec{R}_M)\Psi_n(\vec{R}_1 \dots \vec{R}_M; z_1 \dots z_N)$$

- Multi-quasiparticle wave function has (usually) trivial monodromies (bosonic/ fermionic)
- In general no explicit expressions as integration cannot be performed analytically
- Exception: using particular form of quasielectron and quasihole wave functions → positive Jain CF series (Hansson, MH, Viefers 2009)
  full hierarchy (Suorsa, Viefers, Hansson 2010)

realizes any filling fraction p/q with q odd

properties: (no plasma analogy!)

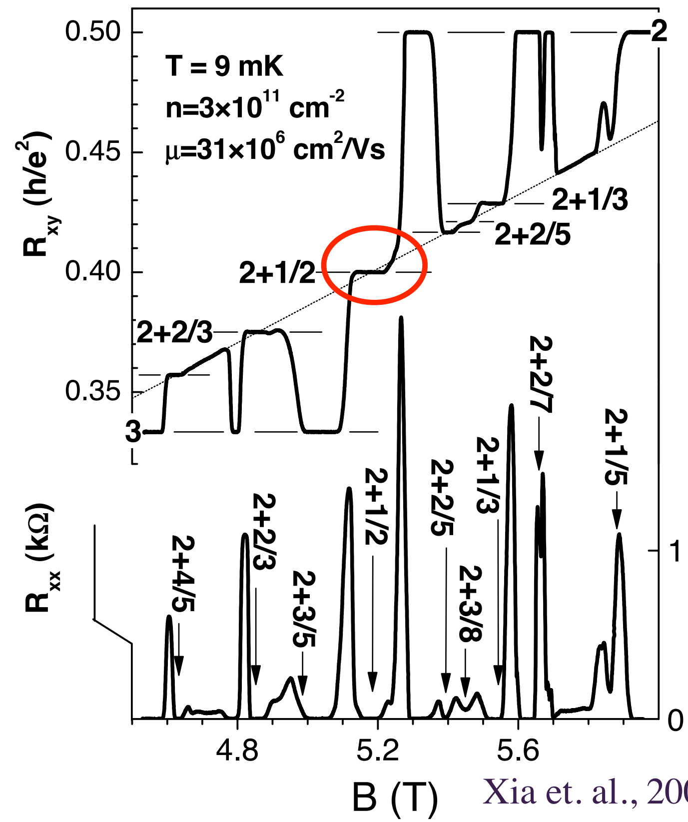
- fractional charge ✔
- relative stability ✔
- screening ✗
- anyonic braiding statistics ✗ (Wen-Zee effective Chern Simons theory)

First excited Landau level

The fractional quantum Hall effect at ν=5/2 was discovered in 1987 (Willet et.al. 1987)

- numerics suggests that the Moore-Read state (or alternatively the anti-Pfaffian) describes this quantum Hall liquid (Morf 1998, Rezayi et al. 2001, Li&Haldane 2008)
- excitations have charge e/4
- believed to be non-Abelian
- other nonabelian candidates:

  7/3, 8/3, 12/5 (Read & Rezayi 1999, Wojs 2001)



Xia et. al., 2004

The Moore-Read idea

Laughlin's wave functions "look like" correlation functions in a Conformal Field Theory. (Fubini, Moore&Read)

- QH wave functions are the conformal blocks of a unitary, rational conformal field theory
- electrons and quasiholes are represented as local operators
- quasihole statistics encoded in the braiding properties of the corresponding operators (if conformal blocks are orthogonal)
- generalization to other FQH states (Moore-Read, Read-Rezayi, NASS, composite fermions, etc.)
- quasielectrons are more complicated (Hansson, MH, Regnault, Viefers 2009)

massless scalar boson field: $\langle\varphi(z)\varphi(w)\rangle = -\ln(z-w)$

particles are described by **vertex** operators: $V_\alpha(z) \quad = \quad e^{i\alpha\varphi(z)}$

$\langle V_\alpha(z)V_\beta(w)\rangle \sim (z-w)^{\alpha\beta} \longrightarrow$ fermionic statistics for α^2 odd integer

Laughlin 1/3: $V(z) = e^{i\sqrt{3}\varphi(z)}$

$$H(w) = e^{\frac{i}{\sqrt{3}}\varphi(w)}$$

The Moore-Read state

using Ising conformal field theory (Moore and Read, 1991)

$$\Psi_{5/2} = \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) \prod_{i<j} (z_i - z_j)^2$$

Ising CFT description

$$\boxed{\begin{aligned} V(z) &= \psi(z) e^{i\sqrt{2}\varphi(z)} \\ H(w) &= \sigma(w) e^{\frac{i}{\sqrt{8}}\varphi(w)} \end{aligned}}$$

$$\psi \times \psi = 1$$
$$\sigma \times \psi = \sigma$$
$$\sigma \times \sigma = 1 + \psi$$

The Moore-Read state

using Ising conformal field theory (Moore and Read, 1991)

$$\Psi_{5/2} = \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right)\prod_{i<j}(z_i - z_j)^2$$

Ising CFT description

$$\boxed{\begin{aligned} V(z) &= \psi(z)e^{i\sqrt{2}\varphi(z)} \\ H(w) &= \sigma(w)e^{\frac{i}{\sqrt{8}}\varphi(w)} \end{aligned}}$$

- Pairing of electrons
- Fundamental quasiholes have charge 1/4
- For $2n$ quasiholes at fixed position there are 2^{n-1} linear independent, degenerate states
- Braiding and quasihole degeneracy are manifest plasma analogy (Read 2009; Bonderson, Gurarie, Nayak 2011)

$$\psi \times \psi = 1$$
$$\sigma \times \psi = \sigma$$
$$\sigma \times \sigma = 1 + \psi$$

The Moore-Read state

using two layer description (Cappelli et al. 2001)

$$\Psi_{5/2} = \mathcal{S}\left[\prod_{\alpha<\beta}(z_\alpha - z_\beta)^2 \prod_{a<b}(z_a - z_b)^2\right]\prod_{i<j}(z_i - z_j)$$

$$\begin{aligned} \alpha, \beta &= 1, \ldots, \frac{N}{2} \\ a, b &= \frac{N}{2}+1, \ldots, N \end{aligned}$$

- Pairing of electrons
- Fundamental quasiholes have charge 1/4
- For $2n$ quasiholes at fixed position there are 2^{n-1} linear independent, degenerate states
- Braiding properties are hidden

Bosonic CFT description

$$\boxed{\begin{aligned} V(z) &= \cos\phi(z)e^{i\sqrt{2}\varphi(z)} \\ H_\pm(\eta) &= e^{\pm i\phi(\eta)/2}e^{\frac{i}{2\sqrt{2}}\varphi(\eta)} \end{aligned}}$$

- Easy generalization to more layers (Read-Rezayi series) and different filling fractions within the layers → nonabelian hierarchy
- Allows construction of quasielectron operator

Nonabelian hierarchy

hierarchy construction using the Moore-Read state as parent state:

$$\Psi_{n+1}(z_1 \ldots z_N) = \int d^2\vec{R}_1 \ldots \int d^2\vec{R}_M \, \Phi^\star(\vec{R}_1 \ldots \vec{R}_M)\Psi_n(\vec{R}_1 \ldots \vec{R}_M; z_1 \ldots z_N)$$

multi-qp wave function not unique

- macroscopic degeneracy
- branch cuts

Nonabelian hierarchy

$$\Psi_{n+1}(z_1 \dots z_N) = \int d^2\vec{R}_1 \dots \int d^2\vec{R}_M \, \Phi^\star(\vec{R}_1 \dots \vec{R}_M) \Psi_n(\vec{R}_1 \dots \vec{R}_M; z_1 \dots z_N)$$

Bonderson & Slingerland (2008):

interactions between Ising anyons lead to energy splitting of fusion channels 1 and ψ
(Baraban et al. 2009)
pairing of Ising anyons → abelian excitations of charge e/2
use abelian pseudo wave function for hierarchical states

nonabelian statistics remains unchanged (Ising anyons)

Example: pairing in 1-channel

$$\Psi_\nu = \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) \Psi_\nu^{(HH)}(z_1, \dots, z_N)$$

'Moore-Read state with altered charge factor'

Nonabelian hierarchy

$$\Psi_{n+1}(z_1 \dots z_N) = \int d^2\vec{R}_1 \dots \int d^2\vec{R}_M \sum_\alpha \Phi^\star_\alpha(\vec{R}_1 \dots \vec{R}_M)\Psi^\alpha_n(\vec{R}_1 \dots \vec{R}_M; z_1 \dots z_M)$$

Levin & Halperin (2009):

use appropriate vector-valued pseudo wave function

$$\Phi^\star_\alpha(\vec{R}_1, \dots, \vec{R}_M) = \langle \prod_{j=1}^{M} \bar{\sigma}(\bar{\eta}_j) e^{i\sqrt{2m+1/8}\,\bar{\phi}(\bar{\eta}_j)} \mathcal{O}_{bg} \rangle_\alpha$$

no branch cuts as full Ising correlators contain both holomorphic and anti-holomorphic pieces (Belavin, Polyakov, Zamolodchikov 1983)

Abelian daughter states

Nonabelian hierarchy

$$\Psi_{n+1}(z_1 \dots z_N) = \int d^2\vec{R}_1 \dots \int d^2\vec{R}_M \, \Phi^\star(\vec{R}_1 \dots \vec{R}_M)\Psi_n(\vec{R}_1 \dots \vec{R}_M; z_1 \dots z_N)$$

Hermanns (2010):

hide statistics in Berry phase by using Cappellis two-layer description
two types of quasiparticles that are not correlated
each has their own pseudo wave function
unique daughter state despite macroscopic degeneracy

$$\Psi_{1/2}(\vec{r}_1 \dots \vec{r}_n) = \mathcal{S}\left[\Psi_{1/2}(\vec{r}_1 \dots \vec{r}_{N/2})\Psi_{1/2}(\vec{r}_{N/2+1} \dots \vec{r}_N)\right]\prod_{i<j}(z_i - z_j)$$

$$\Big\downarrow \text{ qe-condensate with maximal density}$$

$$\Psi_{4/7}(\vec{r}_1 \dots \vec{r}_n) = \mathcal{S}\left[\Psi_{2/3}(\vec{r}_1 \dots \vec{r}_{N/2})\Psi_{2/3}(\vec{r}_{N/2+1} \dots \vec{r}_N)\right]\prod_{i<j}(z_i - z_j)$$

General form of hierarchical states $\Psi_{\tilde{\nu}} = \mathcal{S}\big[\underbrace{\Psi_\nu \Psi_\nu \dots \Psi_\nu}_{k \text{ layers}}\big]\prod_{i<j}(z_i - z_j)$

Properties?

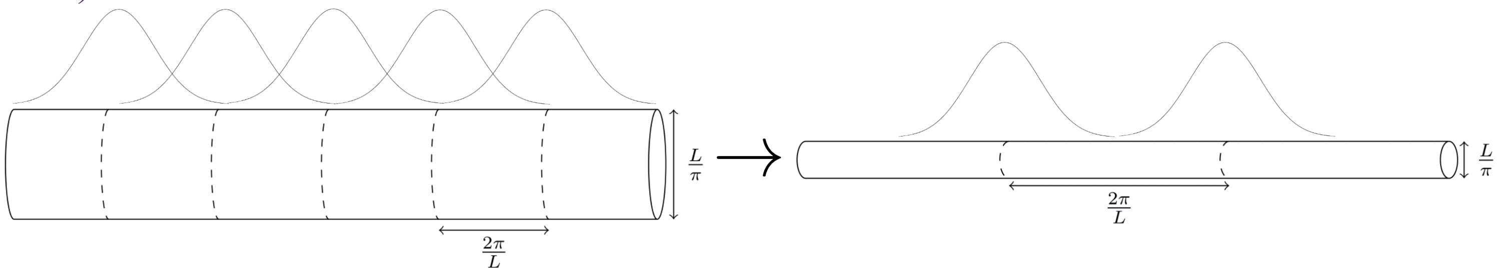
Nonabelian hierarchy

Tournois&Hermanns, in preparation

$$\Psi_{\tilde{\nu}} = \mathcal{S}\big[\underbrace{\Psi_\nu \Psi_\nu \dots \Psi_\nu}_{k \text{ layers}}\big] \prod_{i<j} (z_i - z_j)$$

quasiparticle charge e/q for ν=p/q

SU(q)$_k$ fusion rules can be derived using the thin torus analysis (Ardonne 2009)



surprisingly: many properties retained (ground state degeneracy, fractional charge, domain wall structure, fusion rules)

Nonabelian hierarchy

explicit CFT description possible for filling fractions ν=(q-1)/q (Jain series) using the close relation to generalizations of the nonabelian spin singlet states (Schoutens&Ardonne 1999)

Gepner parafermions (Gepner 1987, Ardonne PhD thesis 2002)

$$\psi_\alpha = \Phi_\alpha^1 \qquad \sigma_\omega = \Phi_\omega^\omega \qquad (\tilde{\alpha}_i, \tilde{\alpha}_j) = 1 + \delta_{i,j} \qquad (\tilde{\alpha}_i, \omega_j) = \delta_{i,j}$$

electron operators: $V_{\tilde{\alpha}_n}(z) = \psi_{\tilde{\alpha}_n}(z) \exp\left[\frac{i}{\sqrt{k}}(\tilde{\alpha}_n \cdot \varphi)(z)\right]$

hole operators: $H_{\omega_n}(w) = \sigma_{\omega_n}(w) \exp\left[\frac{i}{\sqrt{k}}(\omega_n \cdot \varphi)(w)\right]$

Nonabelian hierarchy

explicit CFT description possible for filling fractions ν=(q-1)/q (Jain series) using the close relation to generalizations of the nonabelian spin singlet states (Schoutens&Ardonne 1999)

Gepner parafermions (Gepner 1987, Ardonne PhD thesis 2002)

$$\psi_\alpha = \Phi^1_\alpha \qquad \sigma_\omega = \Phi^\omega_\omega \qquad (\tilde{\alpha}_i, \tilde{\alpha}_j) = 1 + \delta_{i,j} \qquad (\tilde{\alpha}_i, \omega_j) = \delta_{i,j}$$

electron operators: $V_{\tilde{\alpha}_n}(z) = \psi_{\tilde{\alpha}_n}(z) \exp\left[\frac{i}{\sqrt{k}}(\tilde{\alpha}_n \cdot \varphi)(z)\right]$

hole operators: $H_{\omega_n}(w) = \sigma_{\omega_n}(w) \exp\left[\frac{i}{\sqrt{k}}(\omega_n \cdot \varphi)(w)\right]$

spin-polarize state
- introduce derivative (orbital spin)
- symmetrize over 'spin'

Nonabelian hierarchy

explicit CFT description possible for filling fractions ν=(q-1)/q (Jain series) using the close relation to generalizations of the nonabelian spin singlet states (Schoutens&Ardonne 1999)

Gepner parafermions (Gepner 1987, Ardonne PhD thesis 2002)

$$\psi_\alpha = \Phi^1_\alpha \qquad \sigma_\omega = \Phi^\omega_\omega \qquad (\tilde{\alpha}_i, \tilde{\alpha}_j) = 1 + \delta_{i,j} \qquad (\tilde{\alpha}_i, \omega_j) = \delta_{i,j}$$

electron operators: $V_{\tilde{\alpha}_n}(z) = \partial_z^{(n-1)} \psi_{\tilde{\alpha}_n}(z) \exp\left[\frac{i}{\sqrt{k}}(\tilde{\alpha}_n \cdot \varphi)(z)\right]$

hole operators: $H_{\omega_n}(w) = \sigma_{\omega_n}(w) \exp\left[\frac{i}{\sqrt{k}}(\omega_n \cdot \varphi)(w)\right]$

spin-polarize state
- introduce derivative (orbital spin)
- symmetrize over ‘spin’

Quasihole state counting and braiding believed to remain unchanged by symmetrization due to different orbital spin

Open questions & future directions

gain deeper understanding of nonabelian SU(n) singlet states and their braiding properties

verify properties of model wave functions such as

- screening
- quasiparticle braiding statistics

↪MPS description from CFT description (Zaletel & Mong 2012)

effective low energy gauge theories for nonabelian hierarchy states