YANG-MILLS THEORY FROM STRING FIELD THEORY ON D-BRANES

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Abstract We propose the action for string field theory on D(-1)-branes and calculate approximation to the effective action for fields φ^i.

ITEP-TH-38/01

In the last couple of years, renewed attention has been paid to bosonic string field theory [1], in particular due to its application to D-branes (see e. g. [2] and references therein). Connection between D-branes and solutions of string field theory equations of motion [3] plays important role in the study of perturbative open string vacuum [4].

However, the existing formulation of string field theory [5] implies Neumann boundary conditions on the worldsheet. In this note we construct SFT with Dirichlet boundary conditions. We propose the action for string field theory on multiple D(-1)-brane background and provide some calculations of non-abelian effective action for fields φ^i [6] from string field theory, following the programm, initiated in [7].

1. STRING FIELD THEORY ON D(-1)-BRANES

Consider a system of N D(-1)-branes placed in the points with coordinates C^i_a, $i = 0, ..., 25$, $a = 1, ..., N$. Open strings, attached to them, have Dirichlet boundary in all directions.

We propose, that string field theory in this case is described by the following action [1]

$$S=\sum_{a,b}\int \Psi_{ab}*(Q\Psi)_{ba}+\frac{2}{3}g\sum_{a,b,c}\int \Psi_{ab}*\Psi_{bc}*\Psi_{ca}. \qquad (1.1)$$

Here Ψ_{ab} is the string field corresponding to string, that is attached to D(-1)-branes with coordinates C^i_a, C^i_b; Q is the open string BRST operator and $*$ is the string field theory star product.

On the CFT language [8] the action (1.1) can be written as

$$S=\sum_{a,b}\langle g_1\left[\mathcal{O}_{\Psi,ab}(0)\right]g_2\left[Q\mathcal{O}_{\Psi,ba}(0)\right]\rangle+$$

$$+\frac{2}{3}g\sum_{a,b,c}\langle h_1\left[\mathcal{O}_{\Psi,ab}(0)\right]h_2\left[\mathcal{O}_{\Psi,bc}(0)\right]h_3\left[\mathcal{O}_{\Psi,ca}(0)\right]\rangle. \qquad (1.2)$$

Here $\mathcal{O}_{\Psi,ab}(0)$ is the CFT operator (placed in the point $z=0$ on the worldsheet), corresponding to a string field Ψ_{ab}; conformal mappings [8] put operators $\mathcal{O}_{\Psi,ab}(0)$ to the points $g_1(0)=-1, g_2(0)=1$, $h_1(0)=1, h_2(0)=e^{\frac{2\pi i}{3}}, h_3(0)=e^{-\frac{2\pi i}{3}}$ on the boundary of unit disk. Brackets $\langle ...\rangle$ denote correlator of operators on the unit disk with boundary conditions (X^i is the CFT field)

$$X^i(\theta)=\begin{cases} C^i_b, & \text{if } 0<\theta<\pi \\ C^i_a, & \text{if } \pi<\theta<2\pi \end{cases} \qquad (1.3)$$

in the first term in (1.2) and

$$X^i(\theta)=\begin{cases} C^i_b, & \text{if } 0<\theta<\frac{2\pi}{3} \\ C^i_c, & \text{if } \frac{2\pi}{3}<\theta<\frac{4\pi}{3} \\ C^i_a, & \text{if } \frac{4\pi}{3}<\theta<2\pi \end{cases} \qquad (1.4)$$

in the second term. Explicit calculations can be done, using the boundary CFT operator [9]

$$V=e^{\oint X(\theta)\partial_n X(\theta)}, \qquad (1.5)$$

that shifts the Dirichlet boundary conditions.

2. EFFECTIVE ACTION

The string field contains an infinite number of local fields, including states of arbitrarily high spin. Taking into account that it should has ghost number 1, we may expand it up to the level 2

$$\Psi_{ab}=(t_{ab}c_1+\varphi^i_{ab}\alpha^i_{-1}c_1+Fc_0+B^{ik}_{ab}\alpha^i_{-1}\alpha^k_{-1}c_1$$

$$+D^i_{ab}\alpha^i_{-1}c_0+G^i_{ab}\alpha^i_{-2}c_1+\beta_{ab}c_{-1}+\gamma_{ab}c_0b_{-2}c_1+...)|\Omega\rangle. \qquad (1.6)$$

Here $|\Omega\rangle$ is the state corresponding to unit operator [8]. Relation of this state to the usual Hilbert space vacuum is given by $|\Omega\rangle = b_{-1}|0\rangle$. Using the operator-state correspondence, the string field (1.6) may be rewritten as CFT operator.

Our main goal is to find effective action for the fields φ^i. It is expected to be proportional to the Yang-Mills action

$$S_{YM}\propto -\frac{1}{g^2}\mathrm{Tr}\left[g\varphi^i+\frac{C^i}{\sqrt{2\alpha'}\pi},g\varphi^k+\frac{C^k}{\sqrt{2\alpha'}\pi}\right]^2=$$

$$=\frac{1}{\alpha'\pi^2}\sum_{a,b}\left(\Delta C^i_{ab}\Delta C^k_{ba}\varphi^i_{ab}\varphi^k_{ba}+\Delta C^2_{ab}\varphi^k_{ab}\varphi^k_{ba}\right)+$$

$$+4g\mathrm{Tr}[\varphi^i,\varphi^k]\left[\frac{C^k}{\sqrt{2\alpha'}\pi},\varphi^i\right]-g^2\mathrm{Tr}\{\varphi^i,\varphi^k\}^2+4g^2\mathrm{Tr}(\varphi^i\varphi^i)^2, \qquad (1.7)$$

where $\Delta C^i_{ab}=C^i_a-C^i_b$ and symbol Tr means summing over matrix indices a,b. Here we use the identity

$$\mathrm{Tr}[\varphi^i,\varphi^k]^2=\mathrm{Tr}\{\varphi^i,\varphi^k\}^2-4\mathrm{Tr}(\varphi^i\varphi^i)^2. \qquad (1.8)$$

Following the standard prescription [10, 7], we should expand the action (1.2) using the state (1.6) and integrate out all massive fields exept φ^i. The only fields up to the level 2 that contribute to the effective action are t, φ^i, B^{ik}, F and β. Substituting the string field (1.6) into the action (1.1) and keeping the dependence on those fields we get

$$S=\sum_{a,b}\left(\left(\frac{\Delta C^2_{ab}}{4\alpha'\pi^2}-1\right)t_{ab}t_{ba}+\frac{\Delta C^2_{ab}}{4\alpha'\pi^2}\varphi^i_{ab}\varphi^i_{ba}+\right.$$

$$\left.+2\left(\frac{\Delta C^2_{ab}}{4\alpha'\pi^2}+1\right)B^{ik}_{ab}B^{ik}_{ba}-\left(\frac{\Delta C^2_{ab}}{4\alpha'\pi^2}+1\right)\beta_{ab}\beta_{ba}\right)-$$

$$-2\mathrm{Tr}F^2+2\sum_{a,b}\frac{\Delta C^i_{ba}}{\sqrt{2\alpha'}\pi}F_{ab}\varphi^i_{ba}+\frac{2}{3}g\mathrm{Tr}\left(\frac{3}{4}[\varphi^i,\varphi^k]\left[\frac{C^k}{\sqrt{2\alpha'}\pi},\varphi^i\right]-\right.$$

$$\left.-\frac{3^2\sqrt{3}}{2^2}t\varphi^i\varphi^i-\frac{2^3}{\sqrt{3}}B_{ik}\varphi^i\varphi^k-\frac{11}{2^2\sqrt{3}}\beta\varphi^i\varphi^i\right)+... \qquad (1.9)$$

Integrating out fields t, F, B^{ik} and β we obtain the effective action

$$S_{eff}=\frac{1}{4\alpha'\pi^2}\sum_{a,b}\left(\Delta C^i_{ab}\Delta C^k_{ba}\varphi^i_{ab}\varphi^k_{ba}+\Delta C^2_{ab}\varphi^i_{ab}\varphi^i_{ba}\right)+$$

$$+\frac{1}{2}g\mathrm{Tr}[\varphi^i,\varphi^k]\left[\frac{C^k}{\sqrt{2\alpha'}\pi},\varphi^i\right]-\frac{2^3}{3^3}g^2\mathrm{Tr}\{\varphi^i,\varphi^k\}^2+\frac{17\cdot 5^2}{3^3\cdot 2^3}g^2\mathrm{Tr}(\varphi^i\varphi^i)^2 \tag{1.10}$$

We can see, that the structure of first two terms is exactly the same as in Yang-Mills action (1.7). However, the terms of order φ^4 cannot be combined into the squared commutator. In order to get $\mathrm{Tr}[\varphi^i,\varphi^k]^2$ we should add infinitely many fields on higher levels into (1.6). One could also calculate string field theory four-point function. We are leaving this work for future.

Acknowledgments

I am grateful to E. Akhmedov, A. Dymarsky, D. Gal'tsov, A. Konechny, A. Mironov, A. Morozov, N. Nekrasov, V. Pestun, K. Saraikin and especially to A. Gerasimov for useful discussions. I would like to thank the organizers of the Cargèse 2002 ASI for warm and hospitable atmosphere. This work was partly supported by the Russian President's grant 00-15-99296, RFBR grant 01-02-17682 and INTAS grant 00-334.

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