Übungen zur Selbstkontrolle IV

1) Berechnen Sie:

a) $(\frac{3}{-8} - \frac{-11}{12}) \cdot \frac{-8}{13} =$

b) $(\frac{2}{-7} + \frac{-4}{5})(\frac{13}{19} - \frac{1}{2}) =$

c) $-\frac{7}{12} + \frac{3}{4}(\frac{7}{9} - \frac{5}{18}) =$

d) $-\frac{9}{10} \cdot (\frac{4}{9} - \frac{7}{15}) \cdot (\frac{1}{3} - \frac{1}{2}) =$

2) Vereinfachen Sie:

a) $\frac{a^2 - b^2}{a^2 + b^2} \cdot (\frac{a-b}{a+b} + \frac{a+b}{a-b}) =$

b) $\frac{x-y}{x+y} + \frac{y-x}{x-y} =$

c) $\frac{z^2-1}{z+1} - \frac{z^2+1}{z-1} =$

d) $(\frac{1}{(a+b)^2} + \frac{1}{(a-b)^2}) \cdot \frac{a^2-b^2}{a^2+b^2} =$

3) Vereinfachen Sie die folgenden Doppelbrüche:

a) $\frac{\frac{a^2-b^2}{(a+b)^2}}{\frac{a-b}{a+b}} =$

b) $\frac{\frac{x^2-2xy+y^2}{x^2-y^2}}{\frac{(x-y)^2}{x+y}} =$

c) $\frac{\frac{1-z}{z+1}}{1 - \frac{z}{z+1}} =$

d) $\frac{\frac{\frac{a}{b} - \frac{b}{a}}{a^2-b^2}}{\frac{a-b}{a+b}} =$

1b Mathematik (Kg) **12. März 2010**

Übungen zur Selbstkontrolle IV — Lösungen

1) a) $(\frac{3}{-8} - \frac{-11}{12}) \cdot \frac{-8}{13} = (-\frac{3}{8} + \frac{11}{12}) \cdot (-\frac{8}{13}) = \frac{-9+22}{24} \cdot (-\frac{8}{13}) = -\frac{13\cdot 8}{24\cdot 13} = -\frac{1}{3}$

b) $(\frac{2}{-7} + \frac{-4}{5})(\frac{13}{19} - \frac{1}{2}) = \frac{-10-28}{35} \cdot \frac{26-19}{38} = -\frac{38\cdot 7}{35\cdot 38} = -\frac{1}{5}$

c) $-\frac{7}{12} + \frac{3}{4}(\frac{7}{9} - \frac{5}{18}) = -\frac{7}{12} + \frac{3}{4}\cdot\frac{14-5}{18} = -\frac{7}{12} + \frac{3}{4}\cdot\frac{1}{2} = \frac{-14+9}{24} = -\frac{5}{24}$

d) $-\frac{9}{10}\cdot(\frac{4}{9} - \frac{7}{15})\cdot(\frac{1}{3}-\frac{1}{2}) = -\frac{9}{10}\cdot\frac{20-21}{45}\cdot\frac{2-3}{6} = -\frac{9}{10\cdot 45\cdot 6} = -\frac{1}{300}.$

2) a) $\frac{a^2-b^2}{a^2+b^2}\cdot(\frac{a-b}{a+b}+\frac{a+b}{a-b}) = \frac{a^2-b^2}{a^2+b^2}\cdot\frac{(a-b)^2+(a+b)^2}{a^2-b^2} = \frac{a^2-2ab+b^2+a^2+2ab+b^2}{a^2+b^2} = \frac{2a^2+2b^2}{a^2+b^2} = 2.$

b) $\frac{x-y}{x+y}+\frac{y-x}{x-y} = \frac{x-y}{x+y} - 1 = \frac{x-y-(x+y)}{x+y} = -\frac{2y}{x+y}.$

c) $\frac{z^2-1}{z+1} - \frac{z^2+1}{z-1} = \frac{(z-1)(z+1)}{z+1} - \frac{z^2+1}{z-1} = z-1-\frac{z^2+1}{z-1} = \frac{(z-1)^2-(z^2+1)}{z-1} = -\frac{2z}{z-1}.$

d) $(\frac{1}{(a+b)^2}+\frac{1}{(a-b)^2})\cdot\frac{a^2-b^2}{a^2+b^2} = \frac{(a-b)^2+(a+b)^2}{(a+b)^2\cdot(a-b)^2}\cdot\frac{a^2-b^2}{a^2+b^2} = \frac{(2a^2+2b^2)\cdot(a^2-b^2)}{[(a+b)(a-b)]^2\cdot(a^2+b^2)} = \frac{2}{a^2-b^2}.$

3) a) $\frac{\frac{a^2-b^2}{(a+b)^2}}{\frac{a-b}{a+b}} = \frac{(a^2-b^2)\cdot(a+b)}{(a+b)^2\cdot(a-b)} = \frac{(a-b)(a+b)\cdot(a+b)}{(a+b)^2\cdot(a-b)} = 1.$

b) $\frac{\frac{x^2-2xy+y^2}{x^2-y^2}}{\frac{(x-y)^2}{x+y}} = \frac{(x^2-2xy+y^2)\cdot(x+y)}{(x^2-y^2)\cdot(x-y)^2} = \frac{(x-y)^2\cdot(x+y)}{(x-y)(x+y)(x-y)^2} = \frac{1}{x-y}.$

c) $\frac{\frac{1-z}{z+1}}{1-\frac{z}{z+1}} = \frac{\frac{1-z}{z+1}}{\frac{z+1-z}{z+1}} = \frac{(1-z)\cdot(z+1)}{(z+1)} = 1-z.$

d) $\frac{\frac{\frac{a}{b}-\frac{b}{a}}{a^2-b^2}}{\frac{a-b}{a+b}} = \frac{\frac{\frac{a^2-b^2}{ab}}{a^2-b^2}}{\frac{a-b}{a+b}} = \frac{\frac{1}{ab}}{\frac{a-b}{a+b}} = \frac{a+b}{ab(a-b)}.$