Übungen zu Potenzgleichungen

Aus: Lambacher Schweizer, Lehrbuch Klasse 10, S. 42

2 Bestimme die Lösungsmenge.

a) $x^6 = 20$ b) $x^6 = -20$ c) $x^5 = 20$ d) $x^5 = -20$
e) $x^4 = 625$ f) $x^5 + 1024 = 0$ g) $343 + x^3 = 0$ h) $x^5 + 17 = -15$
i) $x^3 + 12 = 39$ k) $x^3 - 23 = -13$ l) $87 + x^5 = 93$ m) $x^3 + 0{,}125 = 0$

3

a) $5x^3 - 20 = 7 - 3x^3$ b) $65 - 53x^2 = 16 + 47x^2$ c) $1{,}2x^5 + 0{,}00243 = 0{,}2x^5$

4 Gib die Lösungen auf zwei Nachkommastellen genau an.

a) $x^3 = 100$ b) $x^5 = 15$ c) $x^4 = 13{,}5$ d) $x^6 = 18{,}2$
e) $x^{-3} = -7{,}5$ f) $1{,}2x^4 = 4{,}9$ g) $0{,}3x^3 = 0{,}68$ h) $23{,}4x^4 = 3{,}8$

5 Bestimme die Lösungsmenge.

a) $(x - 3)^3 = 8$ b) $(2x - 1)^4 = 16$ c) $(0{,}4x + 1)^5 = 243$
d) $(7x - 3)^3 = 216$ e) $(5x - 3)^3 - 8 = 0$ f) $(x + 7{,}3)^4 = 256$
g) $(9 - 5x)^7 - 2 = 0$ h) $(84x - 81)^4 = 81$ i) $(7x - 23)^8 = 10^{-8}$
k) $(10^7 \cdot x - 23)^9 = 10^{18}$ l) $125 \cdot 100^2 = (12 - 0{,}1x)^4$ m) $(6{,}5x - 6{,}5)^6 + 36^3 = 0$

6

a) $x^{\frac{1}{2}} = 11$ b) $x^{\frac{1}{2}} = 7$ c) $x^{\frac{1}{3}} = 8$ d) $x^{\frac{1}{5}} = 1$
e) $\sqrt[3]{2x} = 1$ f) $4 = \sqrt[3]{2x}$ g) $\sqrt[3]{x - 1} = 2$ h) $\sqrt[3]{1 - 2x} = -0{,}1$

7

a) $\sqrt{x^3} = 2$ b) $\sqrt[3]{x^2} = 2$ c) $\sqrt[5]{x^2} = 2$ d) $\sqrt[6]{x^5} = 10^{-5}$
e) $x^{\frac{2}{3}} = 3$ f) $x^{\frac{2}{5}} = 2$ g) $x^{\frac{5}{2}} = 1$ h) $x^{\frac{3}{4}} = 0{,}001$

8

a) $\sqrt[3]{x^2 + 3} - 1 = 0$ b) $\sqrt[4]{x^3 - \sqrt{2}} + 1 = 0$ c) $5 - \sqrt[5]{x^3 - 5} = -238$

9 Berechne die x-Koordinaten der Schnittpunkte der Graphen von g und f.

a) $g: x \mapsto x^3$; $f: x \mapsto 6x$ b) $g: x \mapsto x^7$; $f: x \mapsto 6x^6 + x^5$ c) $g: x \mapsto x^3 + 2x^2$; $f: x \mapsto x^4 + 2x^2$

Lambacher Schweizer Band 10, S. 42 – Ergebnisse

Aufgabe 2:

a) $\mathbb{L} = \{\pm\sqrt[6]{20}\}$, b) $\mathbb{L} = \emptyset$, c) $\mathbb{L} = \{\sqrt[5]{20}\}$, d) $\mathbb{L} = \{-\sqrt[5]{20}\}$,
e) $\mathbb{L} = \{\pm 5\}$, f) $\mathbb{L} = \{-4\}$, g) $\mathbb{L} = \{-7\}$, h) $\mathbb{L} = \{-2\}$,
i) $\mathbb{L} = \{3\}$, k) $\mathbb{L} = \{\sqrt[3]{10}\}$, l) $\mathbb{L} = \{\sqrt[5]{6}\}$, m) $\mathbb{L} = \{-0{,}5\}$.

Aufgabe 3:

a) $5x^3 - 20 = 7 - 3x^3 \iff 8x^3 = 27 \iff x^3 = \frac{27}{8} = \frac{3^2}{2^3} \iff x = \frac{3}{2}$, also $\mathbb{L} = \{\frac{3}{2}\}$,
b) $65 - 53x^2 = 16 + 47x^2 \iff 49 = 100x^2 \iff x^2 = \frac{7^2}{10^2} \iff x = \pm\frac{7}{10}$, also $\mathbb{L} = \{\pm\frac{7}{10}\}$,
c) $1{,}2x^5 + 0{,}00243 = 0{,}2x^5 \iff x^5 = -\frac{243}{10^5} = -\frac{3^5}{10^5} \iff x = -\frac{3}{10}$, also $\mathbb{L} = \{-0{,}3\}$.

Aufgabe 4:

a) Eine Lösung: $\sqrt[3]{100} \approx 4{,}64$, b) eine Lösung: $\sqrt[5]{15} \approx 1{,}72$,
c) zwei Lösungen: $\pm\sqrt[4]{13{,}5} \approx \pm 1{,}92$, d) zwei Lösungen: $\pm\sqrt[6]{18{,}2} \approx \pm 1{,}62$,
e) eine Lösung: $-\sqrt[3]{\frac{1}{7{,}5}} \approx -0{,}51$, f) zwei Lösungen: $\pm\sqrt[4]{\frac{4{,}9}{1{,}2}} \approx \pm 1{,}42$,
g) eine Lösung: $\sqrt[3]{\frac{0{,}68}{0{,}3}} \approx 1{,}31$, h) zwei Lösungen: $\pm\sqrt[4]{\frac{3{,}8}{23{,}4}} \approx \pm 0{,}63$.

Aufgabe 5:

a) $(x-3)^3 = 8 \iff x - 3 = 2 \iff x = 5$, also $\mathbb{L} = \{5\}$,
b) $(2x-1)^4 = 16 \iff 2x - 1 = \pm 2 \iff 2x = 3 \vee 2x = -1 \iff x = \frac{3}{2} \vee x = -\frac{1}{2}$, also $\mathbb{L} = \{\frac{3}{2}\,;\, -\frac{1}{2}\}$,
c) $(0{,}4x+1)^5 = 243 = 3^5 \iff 0{,}4x + 1 = 3 \iff x = 5$, also $\mathbb{L} = \{5\}$,
d) $(7x-3)^3 = 216 = 6^3 \iff 7x - 3 = 6 \iff x = \frac{9}{7}$, also $\mathbb{L} = \{\frac{9}{7}\}$.
e) $(5x-3)^3 - 8 = 0 \iff (5x-3)^3 = 2^3 \iff 5x - 3 = 2 \iff x = 1$, also $\mathbb{L} = \{1\}$,
f) $(x+7{,}3)^4 = 256 = 2^8 = 4^4 \iff x + 7{,}3 = \pm 4 \iff x = -3{,}3 \vee x = -11{,}3$, also $\mathbb{L} = \{-3{,}3\,;\, -11{,}3\}$,
g) $(9-5x)^7 - 2 = 0 \iff (9-5x)^7 = 2 \iff 9 - 5x = \sqrt[7]{2} \iff x = \frac{1}{5}(9 - \sqrt[7]{2}) \approx 1{,}58$, also $\mathbb{L} = \{\frac{1}{5}(9 - \sqrt[7]{2})\}$,
h) $(84x-81)^4 = 81 = 3^4 \iff 84x - 81 = \pm 3 \iff 84x = 84 \vee 84x = 78 \iff x = 1 \vee x = \frac{13}{14}$, also $\mathbb{L} = \{1\,,\, \frac{13}{14}\}$,

Aufgabe 6:

Definitionsbereich und Lösungsmengen der Gleichungen:
a) $\mathbb{D} = [0, \infty[$, $\mathbb{L} = \{121\}$, b) $\mathbb{D} = [0, \infty[$, $\mathbb{L} = \{49\}$,
c) $\mathbb{D} = [0, \infty[$, $\mathbb{L} = \{8^3\} = \{512\}$, d) $\mathbb{D} = [0, \infty[$, $\mathbb{L} = \{1\}$,
e) $\mathbb{D} = \mathbb{R}$, $\mathbb{L} = \{\frac{1}{2}\}$, f) $\mathbb{D} = \mathbb{R}$, $\mathbb{L} = \{32\}$,
g) $\mathbb{D} = \mathbb{R}$, $\mathbb{L} = \{9\}$,
h) $\mathbb{D} = \mathbb{R}$, $\sqrt[3]{1-2x} = -0{,}1 \iff 1 - 2x = -10^{-3} \iff 2x = 1{,}001 \iff x = 0{,}5005$, also $\mathbb{L} = \{0{,}5005\}$.

Aufgabe 7:

Definitionsbereich und Lösungsmengen der Gleichungen:

a) $\mathbb{D} = [0, \infty[$, $\mathbb{L} = \{\sqrt[3]{4}\}$,
b) $\mathbb{D} = \mathbb{R}$, $\mathbb{L} = \{\pm\sqrt{8}\} = \{\pm 2\sqrt{2}\}$,
c) $\mathbb{D} = \mathbb{R}$, $\mathbb{L} = \{\pm\sqrt{32}\} = \{\pm 4\sqrt{2}\}$,
d) $\mathbb{D} = [0, \infty[$, $\mathbb{L} = \{10^{-6}\}$,
e) $\mathbb{D} = [0, \infty[$, $\mathbb{L} = \{3^{\frac{3}{2}}\} = \{3\sqrt{3}\}$,
f) $\mathbb{D} = [0, \infty[$, $\mathbb{L} = \{3^{\frac{5}{2}}\} = \{4\sqrt{2}\}$,
g) $\mathbb{D} = [0, \infty[$, $\mathbb{L} = \{1\}$,
f) $\mathbb{D} = [0, \infty[$, $\mathbb{L} = \{0{,}0001\}$.

Aufgabe 8:

Definitionsbereich und Lösungsmengen der Gleichungen:

a) $\mathbb{D} = \mathbb{R}$, $\sqrt[3]{x^2+3} - 1 = 0 \iff \sqrt[3]{x^2+3} = 1 \iff x^2 + 3 = 1^3 = 1 \iff x^2 = 4 \iff x = \pm 2$, also $\mathbb{L} = \{\pm 2\}$,

b) $\sqrt[4]{x^3 - \sqrt{2}} + 1 = 0$ hat keine Lösung, da $\sqrt[4]{\dots}$ nie negativ, insbesondere nie $= -1$ sein kann, also $\mathbb{L} = \emptyset$,

c) $\mathbb{D} = \mathbb{R}$, $5 - \sqrt[5]{x^3 - 5} = -238 \iff \sqrt[5]{x^3 - 5} = 243 = 3^5 \iff x^3 - 5 = 3^{25} \iff x^3 = 3^{25} + 5 \iff x = \sqrt[3]{3^{25} + 5} = \sqrt[3]{847288609448} \approx 9462{,}6$, also $\mathbb{L} = \{\sqrt[3]{847288609448}\}$.

Aufgabe 9:

a) $f(x) = g(x) \iff x^3 = 6x \iff x(x^2 - 6) = 0 \iff x = 0 \ \vee \ x = \pm\sqrt{6}$,

b) $f(x) = g(x) \iff 6x^6 + x^5 = x^7 \iff 0 = x^7 - 6x^6 - x^5 = x^5(x^2 - 6x - 1) \iff x = 0 \ \vee \ x = 3 \pm \sqrt{9+1} = 3 \pm \sqrt{10}$,

c) $f(x) = g(x) \iff x^4 + 2x^2 = x^3 + 2x^2 \iff 0 = x^4 - x^3 = x^3(x - 1) \iff x = 0 \ \vee \ x = 1$.