

Übungen (3)

Überprüfen Sie Ihr Verständnis und Ihre Kenntnisse der Rechengesetze in \mathbb{Q} an den nachfolgenden Übungsaufgaben.

1) Lösen Sie Klammern auf und fassen Sie zusammen:

- a) $(5x - 6y) - (3x - 7y)$
- b) $(3p - 4q) - (-5p + 7q) + (-9p - 10q)$
- c) $8(7u - v) - 12(4u - 5v)$
- d) $-5r(6k - 7r) - 6k(-5r + 6k)$

2) Schreiben Sie die folgenden Terme als Produkt, indem Sie 'ausklammern':

- a) $4x^2y - 6xy + 12xy^3 - 24x^2y^2$
- b) $3pq^2 + 6p^2q^3 - 6p^3q^2 - 3p^2q^2$
- c) $17a^2b + 7ab - 24a^2b - 14a^3b$

3) Lösen Sie Klammern auf und fassen Sie zusammen:

- a) $(x + y)^2, (x + y)^3, (x + y)^4$
- b) $(a + b + c)^2, (a - b + c)^2$
- c) $(x - 3)^2(x - 4)$
- d) $(3x + 2y - z)^2$

4) Vereinfachen Sie:

- a) $\frac{a^2 - b^2}{a^2 + b^2} \cdot \left(\frac{a - b}{a + b} + \frac{a + b}{a - b} \right) =$
- b) $\frac{x - y}{x + y} + \frac{y - x}{x - y} =$
- c) $\frac{z^2 - 1}{z + 1} - \frac{z^2 + 1}{z - 1} =$
- d) $\left(\frac{1}{(a + b)^2} + \frac{1}{(a - b)^2} \right) \cdot \frac{a^2 - b^2}{a^2 + b^2} =$

5) Vereinfachen Sie die folgenden Doppelbrüche:

- a) $\frac{\frac{a^2 - b^2}{(a + b)^2}}{\frac{a - b}{a + b}} =$
- b) $\frac{\frac{x^2 - 2xy + y^2}{x^2 - y^2}}{\frac{(x - y)^2}{x + y}} =$
- c) $\frac{\frac{1 - z}{z + 1}}{1 - \frac{z}{z + 1}} =$
- d) $\frac{\frac{\frac{a}{b} - \frac{b}{a}}{a^2 - b^2}}{\frac{a - b}{a + b}} =$

6) Berechnen Sie die folgenden Bruchterme:

- a) $\frac{2x - y}{x - 1} + \frac{y - 2x}{x + 1} =$
- b) $\frac{x - y}{y - x} =$
- c) $\frac{(x - a)(x^2 + a^2)(x + a)}{(x^2 - a^2)(x + a)^2} =$
- d) $\frac{1}{x} - \frac{1}{x + 1} =$
- e) $\frac{\frac{x + y}{x - y}}{x^2 - y^2} =$
- f) $\frac{\frac{x + y}{x - y}}{x^2 - y^2} =$
- g) $\frac{x - y}{x + y} + \frac{x + y}{x - y} =$
- h) $\frac{\frac{x^2 - y^2}{x - y}}{\frac{x + y}{x - y}} =$

Übungen (3) — Lösungen

- 1) a) $(5x - 6y) - (3x - 7y) = 2x + y$.
 b) $(3p - 4q) - (-5p + 7q) + (-9p - 10q) =$
 $= 3p - 4q + 5p - 7q - 9p - 10q = -p - 21q$.
 c) $8(7u - v) - 12(4u - 5v) = 56u - 8v - 48u + 60v = 8u + 52v$.
 d) $-5r(6k - 7r) - 6k(-5r + 6k) = -30kr + 35r^2 + 30kr - 36k^2 = 35r^2 - 36k^2$.
- 2) a) $4x^2y - 6xy + 12xy^3 - 24x^2y^2 = 2xy(2x - 3 + 6y^2 - 12xy)$.
 b) $3pq^2 + 6p^2q^3 - 6p^3q^2 - 3p^2q^2 = 3pq^2(1 + 2pq - 2p^2 - p)$.
 c) $17a^2b + 7ab - 24a^2b - 14a^3b = -7a^2b + 7ab - 14a^3b = 7ab(-a + 1 - 2a^2)$.
- 3) a) $(x + y)^2 = x^2 + 2xy + y^2$,
 $(x + y)^3 = (x^2 + 2xy + y^2)(x + y) =$
 $= x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3 = x^3 + 3x^2y + 3xy^2 + y^3$,
 $(x + y)^4 = (x^3 + 3x^2y + 3xy^2 + y^3)(x + y) = \dots =$
 $= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$.
 b) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$,
 $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$.
 c) $(x - 3)^2(x - 4) = (x^2 - 6x + 9)(x - 4) = x^3 - 10x^2 + 33x - 36$.
 d) $(3x + 2y - z)^2 = 9x^2 + 4y^2 + z^2 + 12xy - 6xz - 4yz$.
- 4) a) $\frac{a^2 - b^2}{a^2 + b^2} \cdot \left(\frac{a-b}{a+b} + \frac{a+b}{a-b}\right) = \frac{a^2 - b^2}{a^2 + b^2} \cdot \frac{(a-b)^2 + (a+b)^2}{a^2 - b^2} = \frac{a^2 - 2ab + b^2 + a^2 + 2ab + b^2}{a^2 + b^2} = \frac{2a^2 + 2b^2}{a^2 + b^2} = 2$.
 b) $\frac{x-y}{x+y} + \frac{y-x}{x-y} = \frac{x-y}{x+y} - 1 = \frac{x-y-(x+y)}{x+y} = -\frac{2y}{x+y}$.
 c) $\frac{z^2-1}{z+1} - \frac{z^2+1}{z-1} = \frac{(z-1)(z+1)}{z+1} - \frac{z^2+1}{z-1} = z - 1 - \frac{z^2+1}{z-1} = \frac{(z-1)^2 - (z^2+1)}{z-1} = -\frac{2z}{z-1}$.
 d) $\left(\frac{1}{(a+b)^2} + \frac{1}{(a-b)^2}\right) \cdot \frac{a^2 - b^2}{a^2 + b^2} = \frac{(a-b)^2 + (a+b)^2}{(a+b)^2 \cdot (a-b)^2} \cdot \frac{a^2 - b^2}{a^2 + b^2} = \frac{(2a^2 + 2b^2) \cdot (a^2 - b^2)}{[(a+b)(a-b)]^2 \cdot (a^2 + b^2)} = \frac{2}{a^2 - b^2}$.
- 5) a) $\frac{\frac{a^2 - b^2}{(a+b)^2}}{\frac{a-b}{a+b}} = \frac{(a^2 - b^2) \cdot (a+b)}{(a+b)^2 \cdot (a-b)} = \frac{(a-b)(a+b) \cdot (a+b)}{(a+b)^2 \cdot (a-b)} = 1$.
 b) $\frac{\frac{x^2 - 2xy + y^2}{x^2 - y^2}}{\frac{(x-y)^2}{x+y}} = \frac{(x^2 - 2xy + y^2) \cdot (x+y)}{(x^2 - y^2) \cdot (x-y)^2} = \frac{(x-y)^2 \cdot (x+y)}{(x-y)(x+y)(x-y)^2} = \frac{1}{x-y}$.
 c) $\frac{\frac{1-z}{z+1}}{1 - \frac{z}{z+1}} = \frac{\frac{1-z}{z+1}}{\frac{z+1-z}{z+1}} = \frac{(1-z) \cdot (z+1)}{(z+1)} = 1 - z$.
 d) $\frac{\frac{\frac{a-b}{a}}{a^2 - b^2}}{\frac{a-b}{a+b}} = \frac{\frac{a^2 - b^2}{a^2 - b^2}}{\frac{a-b}{a+b}} = \frac{1}{\frac{a-b}{a+b}} = \frac{a+b}{ab(a-b)}$.
- 6) Ergebnisse:
- a) $\frac{2x - y}{x - 1} + \frac{y - 2x}{x + 1} = \frac{4x - 2y}{x^2 - 1}$ b) $\frac{x - y}{y - x} = -1$
- c) $\frac{(x - a)(x^2 + a^2)(x + a)}{(x^2 - a^2)(x + a)^2} = \frac{x^2 + a^2}{(x + a)^2}$ d) $\frac{1}{x} - \frac{1}{x + 1} = \frac{1}{x^2 + x}$
- e) $\frac{x + y}{\frac{x-y}{x^2 - y^2}} = (x + y)^2$ f) $\frac{\frac{x+y}{x-y}}{x^2 - y^2} = \frac{1}{(x - y)^2}$
- g) $\frac{x - y}{x + y} + \frac{x + y}{x - y} = \frac{2x^2 + 2y^2}{x^2 - y^2}$ h) $\frac{\frac{x^2 - y^2}{x - y}}{\frac{x+y}{x-y}} = x - y$