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Xiaowei Wang: Compactifying moduli spaces of K-stable Fano manifolds

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Gieseker-Mumford

<u>Motivation</u>:

GIT MMP

- dim = 1 curve X $K_X > 0$ $\overline{M}_{g \geq 2}$ DM
  - $K_X = 0$
  - $-K_X > 0$ $\mathbb{P}^1$

Nightone

- dim > 1 $K_X > 0$ <u>GIT Gieseker's</u> MMP KSBA
  - not so canonical $\cup$ sm(X, K_X)
  - $-K_X > 0$? K_X $\mathbb{Q}$-center $\cup$ 0

What kind of obj. $\exists$ cpt moduli

- Berman Guenacia $\forall$ KSBA variety $\exists$ KE

<u>Slogan</u>: $\pm K_X > 0$ canonical cpt'tion
KE - cpt'cation

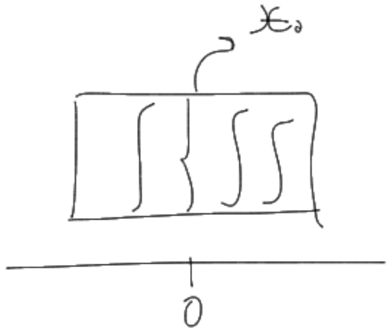
"smoothable" K-semistable Fano variety
admits proper alg. space as
Coarse moduli space

Donaldson–Sun:

- boundedness $\quad$ All $X \subset \mathbb{P}^N$ — uniform
- valuative crit. $\quad \begin{array}{ccc} \mathcal{X}^0 & \subset & \overline{\mathcal{X}} \\ \downarrow & & \downarrow \\ \mathbb{A}^0 & \subset & \mathbb{A} \end{array}$

want
- separatedness $\quad$ filling is unique
- semi stable $\quad$ open cond.

<u>Theorem</u>: $\begin{array}{c} \mathcal{X} \\ \downarrow \\ (C,0) \end{array}$ flat family, sm curve



1) $-K_{\mathcal{X}/C} > 0$

2) $\mathcal{X}_t$ sm for $t \neq 0$, $\mathcal{X}_0$ is $\mathbb{Q}$-Fano

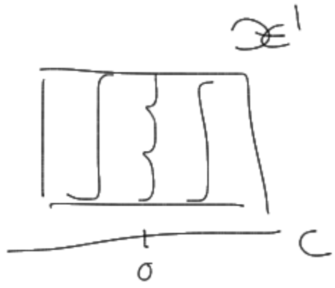
3) $\mathcal{X}_0$ K-poly stable

<u>Then</u>: 1) $\exists \; 0 \in U \subset C$ open s.t. $\mathcal{X}_t$ K-st. $\forall t \in U$

2) $\begin{array}{c} \mathcal{X}' \\ \downarrow \\ C \end{array}$ & $\mathcal{X}'_0$ K poly stable st. $\mathcal{X}'|_{C^0} \cong \mathcal{X}|_{C^0}$

$C^0 = C \setminus \{0\}$

3) $\mathcal{X}$ cont in GH sense



Test configuration

$$\begin{tikzcd} (\mathcal{X},\mathcal{D};\mathcal{L}) \arrow[d] & \supset & (X,D;-rK_X)\times\mathbb{C}^\times \arrow[d] \\ \mathbb{A}^1 & \supset & \mathbb{C}^\times \end{tikzcd}$$

$$DF_\beta(\mathcal{X},\mathcal{D};\mathcal{L}) = DF(\mathcal{X}) + \frac{(1-\beta)}{m}(H(\mathcal{X},\mathcal{D}))$$

$$\beta\text{-K-s.s.} \iff DF_\beta(\mathcal{X},\mathcal{D};\mathcal{L}) \geq 0$$