(n-) cluster tilting modules of self-inj. Nakayama algebras

<u>Def</u> (i) $U \in \operatorname{mod} \Lambda$ <u>n-cluster tilting</u> if

$$\operatorname{add}(U) = \{ M \in \operatorname{mod} \Lambda \mid \operatorname{Ext}^i(M,U) = 0 \ \forall i = 1 \cdots n \}$$
$$= \{ M \in \operatorname{mod} \Lambda \mid \operatorname{Ext}^i(U,M) = 0 \ \forall i = 1 \cdots n \}$$

(ii) Λ is <u>n-rep. finite</u> if $\exists$ n-CT-mod

If Λ : n-RF with gldim $\Lambda \leq n$ then

$$\mathcal{U}_n(\Lambda) = \{ S_n^l(\Lambda) \in \mathcal{D}^b_\Lambda \mid l \in \mathbb{Z} \} \subset \mathcal{D}^b_\Lambda \quad \text{is}$$

n-CT subcat. $\longrightarrow$ $S \circ [-n]$ shifted Serre functor

<u>Theorem</u> (Riedtmann):

T self-inj k-alg. then $\longrightarrow$ $\hat{\Lambda}$ repetitive alg. over Λ

$$T \text{ is } n\text{-RF} \overset{\Leftarrow}{\Longleftrightarrow} \Gamma = \hat{\Lambda} / \varphi$$

where $\Lambda \underset{\text{der}}{\sim} \Lambda'$: n-RF gldim $\Lambda' \overset{=n}{=} 1$

$\varphi : \hat{\Lambda} \longrightarrow \hat{\Lambda}$ admissable autom.

char $k \neq 2$ and $k = \bar{k}$

$$\mathcal{U}_n(\Lambda') \subset \mathcal{D}^b_\Lambda \cong \underline{\operatorname{mod}}\, \Lambda' \circlearrowleft \varphi_*$$

$$\boxed{\varphi_*(\mathcal{U}_n(\Lambda')) = \mathcal{U}_n(\Lambda')}$$

Self-injective Nakayama alg.:

$N_e^{(m)}$:
$$\begin{tikzcd} & 1 \arrow[dr] & \\ e \arrow[ur] & & 2 \arrow[d] \\ & \cdots \arrow[ul] & {} \end{tikzcd} \Big/ I^m$$
I^m ↳ paths of length m

$\Lambda = A_{m-1} \quad 1 \to 2 \to \cdots \to (m-1)$

$\hat{\Lambda}$  $/I^m$

∞-many A_{m-1}

ψ shift

$\varphi = \psi^e$

$N_e^{(m)} \simeq \hat{\Lambda} / \psi^e$

$\varphi_* = \psi_*^e = \tau^e$ Auslander-Reiten translate

$$\begin{tikzcd} \mathcal{D}^b_\Lambda \arrow[r, "\sim"', "\Delta"] \arrow[d, "\pi"'] & \underline{\mathrm{mod}}\, \hat{\Lambda} \arrow[d, "F_*"] \\ \mathcal{C}_n(\Lambda) & \underline{\mathrm{mod}}\, N_e^{(m)} \end{tikzcd}$$

$\mathcal{C}_n(\Lambda) = \mathcal{D}^b_\Lambda / S_n$

n-cluster cat.

basic n-CTO in $\mathcal{C}_n(\Lambda)$

$\updownarrow$

$(n+1)$-angulations of P_N , $N = (n-1)m+2$

$\rho^{\ell(n-1)}$ rotation

Prop. $N_\ell^{(m)}$ is n-RF iff

$$\begin{cases} N \mid 2\ell \\ N \mid t\ell \quad , \; t = \gcd(n+1, 2(m-1)) \end{cases}$$

Exp: $n = m = 3$, $N = 8$