

(n-) cluster tilting modules of self-inj. Nakayama algebras

Def (i) $U \in \text{mod } \Lambda$ n-dual tilting if

$$\begin{aligned} \text{add}(U) &= \{ M \in \text{mod } \Lambda \mid \text{Ext}^i(M, U) = 0 \ \forall i = 1 \dots n \} \\ &= \{ M \in \text{mod } \Lambda \mid \text{Ext}^i(U, M) = 0 \ \forall i = 1 \dots n \} \end{aligned}$$

(ii) Λ is n-rep. finite if \exists n-CT-mod

If Λ : n-RF with $\text{gldim } \Lambda \leq n$ then

$$\mathcal{U}_n(\Lambda) = \{ \mathcal{S}_n^b(\Lambda) \in \mathcal{D}_\Lambda^b \mid l \in \mathbb{Z} \} \subset \mathcal{D}_\Lambda^b \text{ is}$$

n-CT subcat. \rightarrow So [-n] shifted Serre functor

Theorem (Reineke):

T self-inj k -alg. then

$$T \text{ is } n\text{-RF} \iff \Gamma = \hat{\Lambda} / \psi$$

$\hat{\Lambda}$ repetitive alg. over k

where $k \stackrel{\text{der}}{\sim} k'$: n-RF $\text{gldim } k' = 1$

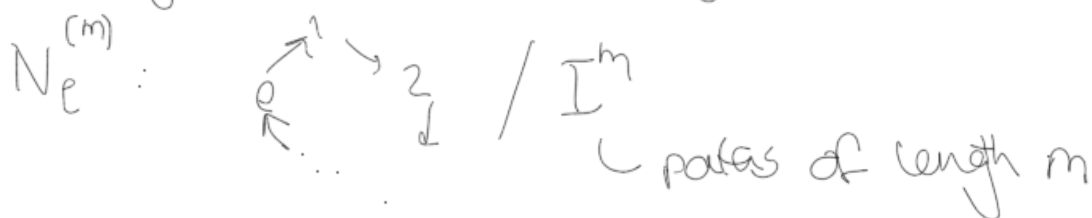
$\psi: \hat{\Lambda} \rightarrow \hat{\Lambda}$ admissible autom.

\hookrightarrow char $k \neq 2$ and $k = \bar{k}$

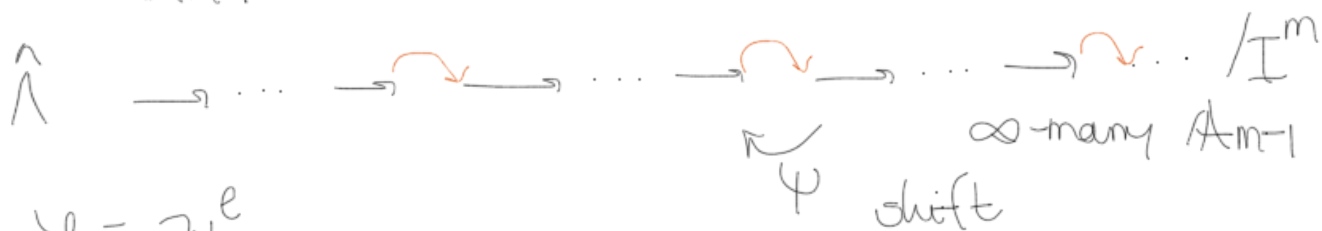
$$\mathcal{U}_n(k') \subset \mathcal{D}_k^0 \cong \text{mod } k' \xrightarrow{\psi_*}$$

$$\psi_*(\mathcal{U}_n(k')) = \mathcal{U}_n(k')$$

Self-injective Nakayama alg.:



$\Lambda = A_{m-1} \quad 1 \rightarrow 2 \rightarrow \dots \rightarrow (m-1)$



$\psi = \tau^e$

$N_e^{(m)} \simeq \hat{\Lambda} / \tau^e$

$\psi_* = \psi_*^l = \tau^e$ Auslander-Reiten translate

$\mathcal{D}_\Lambda^b \xrightarrow[\Delta]{\sim} \underline{\text{mod}} \hat{\Lambda}$
 $\downarrow \tau_*$
 $\underline{\text{mod}} N_e^{(m)}$

$\swarrow \pi$
 $e_n(\Lambda) = \mathcal{D}_\Lambda^b / S_n$
 n-cluster cat.

basic n-CTO in $e_n(\Lambda)$



$(n+1)$ -angulations of P_N , $N = (n+1)m + 2$

$\rho^{l(n-1)}$ rotation

Prop. $N_e^{(m)}$ is n -RF iff

$$\begin{cases} N \mid 2l \\ N \mid te \end{cases}, \quad t = \gcd(n+1, 2(m-1))$$

Exp: $n=m=3$, $N=8$

