LECTURES ON "STRUCTURES IN THE TROPICAL VARIETY OF A G-VARIETY AND REPRESENTATION THEORY"

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Lecture 1: In this first lecture I'll start by recalling part of the fundamental theorem of tropical geometry, which constructs the tropical variety of an affine variety equipped with an embedding as the image of the set of all valuations on the coordinate ring. This means that if we are given a source of valuations we can create and study portions of the tropicalization. The representation theory of a connected reductive group G comes into the picture when we wish to study tropicalizations of varieties equipped with a G action. The set of G-valuations on such a variety is readily describable in terms of the combinatorial elements used in the representation theory of G; this provides a mechanism to create portions of tropical varieties and study them with an established combinatorial language. This basic observation can then be applied to other varieties whose coordinate rings have representation theoretic meaning.

Lecture 2: In this lecture I will continue with the theme of constructing portions of tropical varieties using sets of valuations derived from representation theory. We will need another tool to construct these sets: the connection between higher rank valuations and cones of rank 1 valuations. Varieties equipped with an action by a reductive group G come with a distinguished class of higher rank valuations come from the associated action of the Lie Algebra. I will describe this construction and several associated connections between tropical geometry and branching problems in representation theory.

Lecture 3: In this lecture we'll take a tour through some interesting classes of varieties associated to a connected reductive group G where the tools we've discussed can be applied. Time permitting, I will show examples drawn from the Moduli spaces of vector bundles on curves, spaces of representations of a free group, quiver varieties, and moduli of configurations on flag varieties. We will see how several combinatorial and polyhedral objects from representation theory emerge naturally from the tropical geometry of these examples.