Lectures on "Quivers, quantum groups, and degenerations"

Markus Reineke

We start with the definition of quivers and their representations, and then develop the representation theory of Dynkin quivers, including Gabriel's theorem and basics of Auslander-Reiten theory.

We then construct the Hall algebra of a Dynkin quiver and realize (the positive part of) quantized enveloping algebras of semisimple Lie algebras in terms of quiver representations.

This will then allow us to apply the "K-theory" of Dynkin quivers to the construction of certain degree functions on quantized enveloping algebras. With the aid of these functions, we will construct PBW degenerations and toric degenerations of flag varieties.