Spring School: Tropical Geometry meets Representation Theory

Talks

Fatemeh Mohammadi (University of Bristol)

Toric degenerations of Grassmannians from matching fields

I will talk about the monomial degenerations of the Grassmannian of 3-planes. I will present the necessary condition to obtain a toric initial ideal in terms of the combinatorics of some tropical hyperplane arrangements. This is based on a joint work with Kristin Shaw.

Christoph Pegel (Universität Hannover)

A Continuous Family of Marked Poset Polytopes

Marked poset polytopes appear in representation theory as Gelfand-Tsetlin and FFLV-polytopes: lattice polytopes enumerating bases for irreducible representations of GL_n as well as bases for the homogeneous coordinate rings of flag varieties and their degenerations. In this talk we present a continuous family of polytopes interpolating marked order and marked chain polytopes that can be studied using a subdivsion obtained by intersecting with a tropical hyperplane arrangement.

Igor Makhlin(Skolkovo Institute of Science and Technology)

Degenerate representations and maximal cones in tropical flag varieties

We define weighted PBW degenerations of irreducible $\mathfrak{sl}_n$-representations and partial flag varieties by attaching an integer weight to every negative root. When the chosen weight system lies within a certain polyhedral cone, these objects generalize several known constructions and exhibit fascinating algebraic, geometric and combinatorial properties. In particular, this cone of weight systems can be viewed as a maximal cone in the tropical flag variety, i.e. for every n we obtain an explicitly described maximal cone in the tropical flag variety $\mathcal{F}l_n$. (Joint work with X. Fang, E. Feigin and G. Fourier.)

Daping Weng (Yale University)

Cyclic Sieving Phenomenon of Plane Partitions and Cluster Duality of Grassmannians

Fix two positive integers a and b. Scott showed that a homogeneous coordinate ring of the Grassmannian $Gr_{a,a+b}$ has the structure of a cluster algebra. This homogeneous coordinate ring can be decomposed into a direct sum of irreducible representations of GL_{a+b} which correspond to integer multiples of the fundamental weight w_a. By proving the Fock-Goncharov cluster duality conjecture for the Grassmannian using a sufficient condition found by Gross, Hacking, Keel, and Kontsevich, we obtain bases parametrized by plane partitions for these irreducible representations. As an application we use these bases to show a cyclic sieving phenomenon of plane partitions under a certain sequence of toggling operations. This is joint work with Jiuzu Hong and Linhui Shen.

Veronika Wanner (Universität Regensburg)

Poincaré duality for the tropical Dolbeaut cohomology of non-archimedean Mumford curves

Berkovich analytic spaces are analogues over non-archimedean fields of complex analytic spaces. Recently, Chambert-Loir and Ducros introduced bigraded real-valued differential forms on Berkovich analytic spaces. These are defined using superforms in the sense of Lagerberg on tropical varieties. Pulling back these forms along the tropicalization map, one obtains differential forms on Berkovich spaces. I will give an introduction to the theory of these forms and present some results on their cohomology. In particular, I will talk about recent joint work with P. Jell, where we calculate this cohomology for the projective line and Mumford curves. The key statement here is Poincaré duality which we prove via a local consideration using tropical methods and results.

Paolo Tripoli(University of Nottingham)

Tropical Chow Hypersurfaces

Given a projective variety X, the Chow hypersurface Z_X parametrizes linear spaces that intersect X. Chow hypersurfaces are the basic brick for the construction of a classical moduli spaces called Chow variety. In this talk, I will introduce a tropical Chow hypersurface $\mathrm{trop}(Z_X)$ and I will explore some of its geometric properties. I will show that this object only depends on the tropical variety $\mathrm{trop}(X)$ and I will provide an explicit way to obtain $\mathrm{trop}(Z_X)$ from $\mathrm{trop}(X)$. In the end, I will discuss how tropical Chow hypersurfaces could lead to the construction of a moduli space of tropical varieties.

Marvin Anas Hahn (Universität Tübingen)

Piecewise polynomiality properties of Hurwitz-type counts

Hurwitz numbers count branched genus g, degree d coverings of the Riemann sphere with fixed ramification data. These enumerative objects are deeply connected to Gromov-Witten theory. In recent years several related notions of Hurwitz-type counts and combinatorial interpolations between them appeared in the literature and the uncovering the connections between these counts and Gromov Witten theory – similar to the classical case – has become an active field of research. One of the crucial necessary proporties for such connections to exist is a (piecewise) polynomial behaviour of the Hurwitz-type counts in the initial ramification data. This talk is centered around the polynomial structure and wall-crossing behaviour of interpolations between double, monotone double and Grothendieck dessin d'enfants double Hurwitz numbers. We study this structure by means of tropical geometry and representation theory. Parts of this talk are based on joint work with Reinier Kramer and Danilo Lewanski.

Ben Smith (Queen Mary University of London)

Tropical Oriented Matroids and their Bijections

Oriented matroids are combinatorial objects that encode data arising from hyperplane arrangements. Tropical oriented matroids, the tropical analgoue arising from tropical hyperplane arrangements with full support, were similarly defined by Ardilla and Develin. They have a rich combinatorial structure with one-to-one correspondences to multiple polyhedral and graph theoretical objects. In this talk we shall examine these correspondences and their links to other areas of algebra and combinatorics. We'll also investigate whether we can relax the axioms of tropical oriented matroids to allow for tropical hyperplane arrangements without full support.

Tim Seynnaeve (MPI MiS Leipzig)

Representation theory and fast matrix multiplication

Determining the algorithmic complexity of matrix multiplication is one of the central open problems in computer science. This problem can be shown to be equivalent to determining the Waring rank of the *symmetrized matrix multiplication tensor* $SM_n \in S^3(\mathfrak{sl}_n^*)$. Motivated by this, we study the plethysm $S^k(\mathfrak{sl}_n)$ of the adjoint representation $\mathfrak{sl}_n$ of the Lie group SL_n. This talk is based on a current work in progress with Mateusz Michalek.

Alfredo Nájera (UNAM Oaxaca)

Tropicalisation of cluster varieties and root systems

Cluster varieties are a family of schemes endowed with a positive atlas of tori in the sense of Fock and Goncharov. The space of real tropical points of a cluster variety admits a wall and chamber structure that encodes valuable information of the corresponding scheme. In this talk I will recall the basics of cluster varieties and explain how root systems of Kac-Moody Lie algebras can be used to understand these wall and chamber structures.

Man Wai Cheung (Harvard University)

Quiver representations and theta functions

Scattering diagrams theta functions and broken lines were developed in order to describe toric degenerations of Calabi-Yau varieties and construct mirror pairs. Later, Gross-Hacking-Keel-Kontsevich unravel the relation of those objects with cluster algebras. In the talk, we will discuss how we can combine the representation theory with these objects. We will also see how the broken lines on scattering diagram give a stratification of quiver Grassmannians using this setting.

Matthew Pressland (Universität Stuttgart)

The Caldero–Chapoton formula as a dimer partition function

Certain elements of a Grassmannian cluster algebra, the twisted Plücker coordinates, are expressible as 'dimer partition functions', i.e. weighted sums over the set of perfect matchings of a bipartite graph, or dimer model, in the disk, via the Marsh–Scott formula. Using the categorification of this cluster algebra by Jensen, King and Su, together with the Caldero–Chapton formula, one can also give a formula for the twisted Plücker coordinates in terms of homological data in the category. I will explain how these two formulae are essentially the same, and in so doing relate combinatorial data from the dimer model to homological data in the JKS categorification. For example, a perfect matching on the dimer model encodes a module for the corresponding Jacobian algebra. This is joint work with Ilke Çanakçi and Alastair King.

Timothy Magee (UNAM Oaxaca)

Canonical bases through mirror symmetry

Canonical bases on spaces of sections of line bundles are a central piece of the log Calabi-Yau mirror symmetry program being developed by Gross, Hacking, Keel, Siebert, etc. These bases connect the geometric program to representation theoretic questions. Basis elements correspond to integral tropical points of the mirror log CY. The technology involved tends to be souped-up versions of something from the world of toric varieties, where the picture is far more down to earth and digestible. I'll focus on toric varieties to give a rough feel for how the program works, and give a couple non-toric, but still manageable, examples of representation theoretic interest.

Andrea Maiorana (SISSA Trieste)

Moduli of semistable sheaves as quiver moduli

In the 1980's Drézet and Le Potier realized moduli spaces of Gieseker semistable sheaves on $\mathbb{P}^2$ as what are now called quiver moduli spaces. I will discuss how this can be understood using t-structures and exceptional collections, and how it can be extended to a similar result on $\mathbb{P}^1 \times \mathbb{P}^1$. This construction can be used to prove easily some of the geometric properties of the moduli space of sheaves, and to do some explicit computations.

Philipp Jell (Georgia Institute of Technology)

Lefschetz (1,1)-theorem in tropical geometry

For a complex smooth projective variety X, the classical Lefschetz $(1,1)$-theorem characterizes the classes in $H^{1,1}(X)$ which are Chern classes of holomorphic line bundles.

Combined with duality it implies the Hodge conjecture for divisors. We will discuss a tropical analogue of this. For a tropical manifold X, our tropical Lefschetz $(1,1)$-theorem discribes the classes in the $(1,1)$-tropical cohomolgy which appear as Chern classes of tropical line bundles. As in the classical case this establishes a tropical Hodge conjecture for divisors when combined with Poincaré duality.

Tom Sutherland (Universität Mainz)

Theta functions on moduli spaces of local systems

We consider the so-called theta basis of the ring of regular functions on moduli spaces of $\mathrm{SL}(2, C)$-local systems on Riemann surfaces with (possibly irregular) singularities, whose structure coefficients come from the tropical geometry of the moduli space which here are given by the representation theory of an associated quiver. We will focus on the lowest dimensional examples of such moduli spaces for which three elements of the theta basis which compute the trace of the monodromy of the local systems around particular loops on the Riemann surface embed the moduli space as an affine cubic surface in C^3. In these cases the coefficients of the defining cubic equation arise from the representation theory of a finite, affine or elliptic Dynkin quiver.

Hipolito Treffinger (University of Leicester)

The wall and chamber structure of an algebra

Following the construction of Bridgeland's scattering diagrams, we use King's stability conditions to introduce the wall and chamber structure of a finite dimensional algebra. Later we show how it can be described by g-vectors. If there is enough time, we will give a dual description using c-vectors.