List of errors/corrections for 'Brownian motion'

Version of 26/09/12

- Page 61: In Exercise 2.9 the nonempty, closed set should be a subset of a complete metric space. In the proof, completeness can be used to show that if each ball in a sequence of nested balls with radius tending to zero intersects the set, then the intersection of these balls intersects the set. (Found by *Perla Sousi*).

- Page 65: In Theorem 3.2 (ii) and (iii) one should ask for the *closure* of $\mathcal{B}(x, r)$ to be contained in U, not just the open ball itself. (Found by *Adam Jones*).

- Page 88: In the proof of Theorem 3.48, the Brownian motion started at y could hit the set A even if the Brownian motion started at x does not. Multiplying the middle and right hand side in the first display of this proof by 2 yields a correct inequality, and does not affect the rest of the proof. (Found by *Perla Sousi*).

- Page 98: If one wants to include $\alpha = 0$ into the definition of α-values, Hausdorff content and measure, it is important to use the convention $|\emptyset|^0 = 0$. (Found by *Klaus Schürger*).

- Page 105: In Line 5 $\bigcup_{k=M+1}^{m}$ should read $\bigcup_{k=m}^{M-1}$. Also in (4.4) replace $\mathfrak{C}$ by $\mathfrak{E}$ twice. (Found by *Klaus Schürger*).

- Page 112: The empty set needs to be added to the collection $\mathcal{C}(\partial T)$ to form a semi-algebra, and we need to set $\tilde{\nu}(\emptyset) = 0$. (Found by *Klaus Schürger*).

- Page 154: In Lemma 6.3 during a downcrossing of $[a, b]$ the implicit conditioning on not hitting level c has not been taken into account. As a result in the representation of D_u the mean of the geometric variables Y_j is now $\frac{(c-m)(b-a)}{(c-a)(b-m)}$. As a result in Lemma 6.4 we now get that $\{2\frac{b_n-a_n}{b-a_n} D(a_n, b_n, T) \colon n \in \mathbb{N}\}$ is a submartingale, and this suffices to prove Lemma 6.5. (Found by *Svante Janson*).

- Page 166: The error on Page 154 also has a knock-on effect on Lemma 6.22, where he have to ensure that the stopping takes place at a sufficiently high level, and consequently we obtain Lemma 6.23 only for $a \in (\eta N, \eta N)$ for some $\eta < 1$. This is sufficient for the proof of the main result, Theorem 6.19. (Found by *Svante Janson*).

- Page 195: In the proof of Theorem 7.11 we claim that $\int_s^t H_n(u) dB_u$ is independent of $\mathcal{F}_s$. This is not true in general, but it is true that the conditional expectation of the integral given $\mathcal{F}_s$ is zero, and nothing more is used. (Found by *Nina Gantert*).

- Page 195: In the proof of Theorem 7.11 Doob's maximal inequality is applied to a martingale which is not known to be continuous. As a remedy one can replace $H(s)$ in the given argument by $H_m(s)$. Stochastic integrals of step functions are continuous, by their definition. Since we know that $H_n - H_m$ tends to zero in $\mathbf{L}^2[0, t_0]$ as $n, m \to \infty$, Doob's maximal inequality shows that the stochastic integrals of H_n and H_m are uniformly close, i.e. the difference tends to zero uniformly on $[0, t_0]$. Therefore $X_n = (\int_0^t H_n(s)\, dB(s) \colon 0 \le t \le t_0)$ defines a Cauchy sequence in the space of continuous functions on $[0, t_0]$ in the sup norm. Hence it converges to a continuous limit uniformly, and this limit is the desired continuous version of the stochastic integral. (Found by *Philippe Charmoy*).

- Page 214: In the proof of Theorem 7.43 taking the derivatives inside the double integral is not justified. (Found by *Timo Seppäläinen*).

- Page 230: The multiplicative constant $c(d)$ (introduced in Theorem 3.33) is missing on the left and right hand side of the statement in Corollary 8.12. (Found by *Perla Sousi*).

- Page 261: In the proof of Lemma 9.11 the last displayed formula uses countable stability of Hausdorff dimension and should be stated with $> \gamma$ replacing $\ge \gamma$. The given argument does not transfer 'completely analogously' to the second case, but here is an argument that does: Let $q(t)$ be the probability of $\dim S(\infty) > \gamma$ for Brownian motions started randomly with the law of $B_t^1, ..., B_t^p$. By scaling invariance $q(t)$ is independent of t and therefore equal to zero or one, by the zero-one law for tail events. Hence the given probability is equal to zero or one for Lebesgue-almost all starting points and the last step of the current proof allows to extend this to arbitrary starting configurations. (Found and corrected by *Achim Klenke*).

- Page 271: Throughout the proof of Theorem 9.18 the notation $v \le x$ (where $v \in \Upsilon$ and $x \in \partial\Upsilon$) should be replaced by $v \in x$, and T by Υ. (Found by *Perla Sousi*).

- Page 273: In the second and fourth display, do not subtract $B(\alpha_i)$ from the bridges. (Found by *Perla Sousi*).

- Page 280: In Line 8 insert 'locally' before α-Hölder. (Found by *Klaus Schürger*).

- Page 281 In Line 15 ε needs to be replaced by $\varepsilon d^{-\beta/2}$. (Found by *Klaus Schürger*).

- Page 282 In (9.13) k on the right needs to replaced by $k-1$. This does not affect usability of the lemma. (Found by *Klaus Schürger*).

- Page 284: In the proof of Theorem 9.36 the last equality at the bottom of the page is incorrect. Conditioning on T yields extra information for the percolation set, so that the strong Markov property of Brownian motion cannot be applied. (Found by *Perla Sousi*).

- Page 286: In Exercise 9.7 one should assume that A_1 and A_2 are disjoint and closed. (Found by *Perla Sousi*).

- Page 367: In the last line of the sketch for Exercise 2.7(b) replace $\mathbb{E}[B(T)]$ by $\mathbb{E}|B(T)|$. (Found by *Adam Jones*).

- Page 383: Progress on Problem 1(b) has been achieved by Antunović, Peres and Vermesi, see their preprint in `arXiv:1003.0228`. Progress on Problem 1(c) is in Antunovic, Burdzy, Peres and Ruscher `arXiv:1009.3603` and on Problem 1(d) in Peres and Sousi `arXiv:1010.2987` which also has a relevant zero-one law for 1(b).

- Page 384: Progress on Problem 5 has been achieved by Cammarota and Mörters, see their paper in Electronic Communications in Probability.

- Page 385: A clarification for Problem 7: For the points x in question there should exist times $0 < t_1 < t_2 < t_3$ such that $x = B(t_1) = B(t_2) = B(t_3)$ and x is on the outer boundary of $B[0, t_3]$. Do such points exist?