Robustness of spatial preferential attachment networks

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joint work with

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Preferential attachment

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Bollobás and Riordan defined a growing family of random graphs based on the preferential attachment paradigm and rigorously verified several of the conjectured emerging features.

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A plausible reason for the clustering in networks is the presence of individual features of the nodes such that similarity of features is an additional incentive to form links. We therefore propose a model in which preferential attachment is combined with spatial structure to address this. Similar models have been set up by Flaxman, Frieze and Vera (2006) and Aiello, Bonato, Cooper, Janssen and Pralat (2009).

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- A node born at time t is connected by an edge to each existing node independently with probability

$$\varphi\Big(\frac{t\varrho}{f(k)}\Big),$$

where

  - k is the indegree of the older node at time t,
  - ϱ is the distance of the nodes,
  - $\varphi\colon [0,\infty) \to [0,1]$ is a decreasing profile function,
  - $f\colon \mathbb{N}_0 \to (0,\infty)$ is an increasing attachment rule.

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Conventions:

- We normalise φ so that $\int \varphi(|x|)\, dx = 1$.
- We assume $\gamma := \lim_{k\to\infty} \frac{f(k)}{k}$ exists and $0 < \gamma < 1$.

Suppose the graph G_{t-} is given, and a vertex is born at time t. Then, for each vertex in G_{t-} with indegree k, the probability that it is linked to the newborn vertex is equal to

$$\int \varphi\left(\frac{td(0,y)}{f(k)}\right)\,dy = \frac{f(k)}{t}\,2\int_0^{\frac{t}{2f(k)}} \varphi(x)\,dx \sim \frac{f(k)}{t}.$$

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Indegree processes of different vertices are dependent.

Empirical degree distribution

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Theorem 1 Jacob and M (2013)

The empirical indegree distributions μ_t converge in probability to a deterministic probability measure μ given by

$$\mu(k) = \frac{1}{1+f(k)} \prod_{\ell=0}^{k-1} \frac{f(\ell)}{1+f(\ell)} = k^{-(1+\frac{1}{\gamma})+o(1)},$$

i.e. the asymptotic indegree distribution is a power law with exponent

$$\tau = 1 + \tfrac{1}{\gamma}.$$

The empirical outdegree distribution converges to a light-tailed distribution, which does not influence the power law exponent.

For a finite graph G we define the local clustering coefficient

$$c^{\mathrm{loc}}(v) := \frac{\#\text{triangles containing } v}{\#\text{adjacent edge pairs meeting at } v}$$

and the average clustering coefficient as

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Theorem 2 Jacob and M (2013)

The network $(G_t)_{t>0}$ is clustering in the sense that

$$c^{\text{av}}(G_t) \to c^{\text{av}}_{\infty} > 0 \qquad \text{in probability.}$$

Suppose φ is regularly varying at infinity with index $-\delta$, for $\delta > 1$. The parameter δ controls the probability of long edges and quantifies the clustering; the bigger δ the stronger the clustering.

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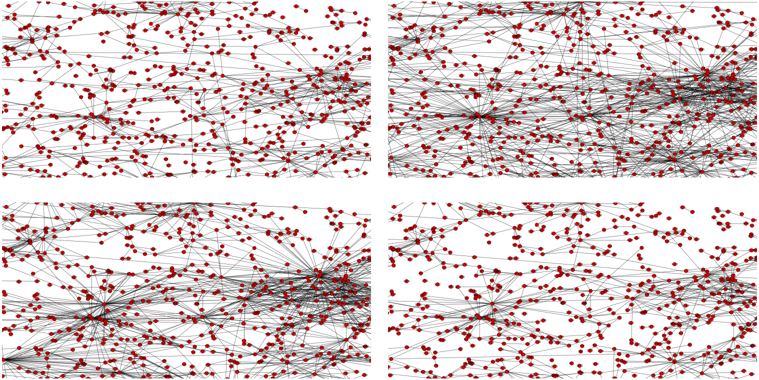

Simulations with parameters (clockwise from top left) (a) $\gamma = 0.5$, $\delta = 2.5$, (b) $\gamma = 0.75$, $\delta = 2.5$, (c) $\gamma = 0.5$, $\delta = 5$, (d) $\gamma = 0.75$, $\delta = 5$.

We now address the problem of robustness of the network $(G_t)_{t>0}$ under percolation. Let $C_t \subset G_t$ be the largest connected component in G_t and denote by $|C_t|$ its size. We say that the network has a giant component if C_t is of linear size or, more precisely, if

$$\lim_{\varepsilon \downarrow 0} \limsup_{t \to \infty} \mathbb{P}\left(\frac{|C_t|}{t} \leq \varepsilon \right) = 0.$$

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We write ${}^p G_t$ for the random subgraph of G_t obtained by Bernoulli percolation with retention parameter $p > 0$ on the vertices of G_t. The network $(G_t)_{t>0}$ is said to be robust if, for any $p > 0$, the network $({}^p G_t)_{t>0}$ has a giant component.

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The network $(G_t)_{t>0}$ is robust if $\gamma > \frac{\delta}{1+\delta}$ or equivalently $\tau < 2 + \frac{1}{\delta}$.

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- Robustness has also been shown by Deijfen, van der Hofstad and Hooghiemstra (2013) for a scale-free long range percolation model.

Let $C_t \subset G_t$ be the largest connected component in G_t. We say that the network has no giant component if C_t has sublinear size, i.e.

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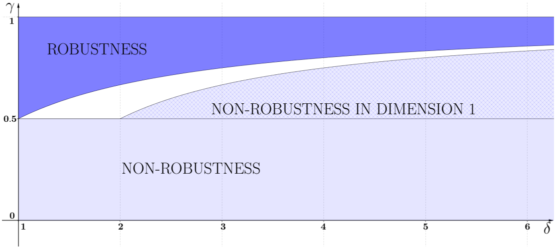

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- Robustness does not occur if the profile function φ is not heavy-tailed, i.e. as $\delta \to \infty$.
- We conjecture non-robustness for $\gamma < \frac{\delta}{\delta+1}$ or equivalently $\tau > 2 + \frac{1}{\delta}$.

Proof strategy for non-robustness

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- The main problem is that, if n is large, there is no easy upper bound for the probability that n distinct vertices $x_1, \ldots, x_n$ in G^∞ form a path $\{x_1 \leftrightarrow x_2 \leftrightarrow \cdots \leftrightarrow x_n\}$.

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- The BK-inequality of van den Berg and Kesten (1985) states that the probability of increasing events occurring disjointly is bounded by the probability of their product.

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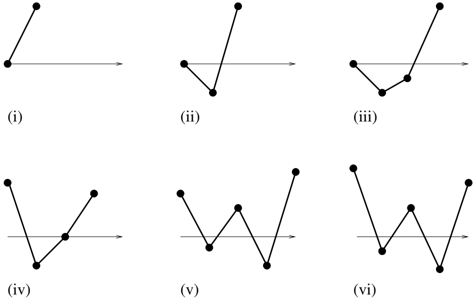
- With this trick the path can be split into pieces which up to symmetry are one of the following six types.



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Now any vertex $z_1' \in G^\infty$ younger than x_0 and x_2 but older than z_1 can only influence the indegree of either z_0 or z_2 at the birth time of z_1, but never both. Hence $z_0 \leftrightarrow z_1 \leftrightarrow z_2$ being a quick path implies the disjoint occurrence of the events $\{z_0 \leftrightarrow z_1\}$ and $\{z_1 \leftrightarrow z_2\}$.

The construction of quick paths is not using spatial information and gives non-robustness only in the case $\tau > 3$. To show non-robustness in the case $\tau > 2 + \frac{1}{\delta - 1}$ a refinement is needed.

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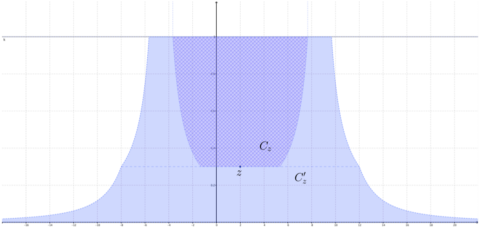
For a point z define a region around z and show that the typical number of vertices outside this region that are connected to z, or any other vertex in C_z, is small. Including only edges that straddle the boundary of the region in a reduced quick path improves the bound if $\delta > 2$.



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- The phase transition between the robust and non-robust phase does not occur at $\tau = 3$, but at a smaller value depending on the clustering strength. This is a new phenomenon.
- The proof of non-robustness is based on an upper bound for the probability of a special class of paths in the network, called the quick paths. We use disjoint occurence of events and the BK inequality to break up paths into short bits whose probabilities can be estimated.

Talks based on

- Spatial preferential attachment networks:
  Power laws and clustering coefficients
  Emmanuel Jacob and Peter Mörters.
  Annals of Applied Probability. 25 (2015) 632-662.

- Robustness of scale-free spatial networks
  Emmanuel Jacob and Peter Mörters.
  Submitted for publication, arXiv:1504.00618

Thank you very much for your attention!