Dr. Nikhil Savale

Vorlesung SUB-RIEMANNIAN SPECTRAL GEOMETRY (14722.0036)

Mo., Mi. 10-11.30 Uhr
im Übungsraum 2, Gyrhofstraße
Bereich: Geometrie und Topologie, Analysis
Belegungsmöglichkeiten:
Mathematik: Master

Übungen SUB-RIEMANNIAN SPECTRAL GEOMETRY (14722.0037)

Mi. 14-15.30 Uhr
im Übungsraum 2, Gyrhofstraße

Sub-Riemannian (sR) geometry, a generalization of Riemannian geometry, is the study of bracket generating metric-distributions inside the tangent space of manifold. There is a wealth of examples of such distributions including contact/even-contact hyper planes, Martinet, Grushin and Engel distributions. The subject arose out of several motivations from problems in classical and quantum mechanics, thermodynamics, hypoelliptic PDE's, calculus of variations, optimal control/transport and many more.

Although analogous in definition to Riemannian geometry, the geometry of sub-Riemannian manifolds presents several new and interesting features. One of these includes the Hausdorff dimension, given by the volume growth rate of metric balls, and which in general is strictly bigger than the topological dimension of the manifold. Another includes the phenomenon of abnormal geodesics which do not satisfy any variational equations. Furthermore, the Laplacian of a sub-Riemannian manifold is a hypoelliptic operator, being a sum of squares operator of Hörmander type. Spectral asymptotics and microlocal questions for the sR Laplacian such as Weyl's laws, wave trace expansion, quantum ergodicity and propagation of singularities and control/observability for its wave equation are largely unexplored and a topic of active research.

The purpose of the course will be to give an introduction to sub-Riemannian geometry and the spectral theory of its Laplacian. The first half of the course will cover the geometric/dynamical aspects [1,3] including Hausdorff dimension, distance/volume comparisons, characterization and examples of abnormal geodesics. The second half will be devoted to the sub-Riemannian Laplacian with the main objectives being the proof of its hypoellipticity [4] and small time heat kernel expansion. Time permitting we will explore connections to Bergman-Szego kernel expansion and estimates on CR and complex manifolds [2].

Literatur

[1] M. Gromov, Carnot-Caratheodory spaces seen from within, in Sub-Riemannian geometry, vol. 144 of Progr. Math., Birkhäuser, Basel, 1996, pp. 79–323.

[2] G. Marinescu and N. Savale, Bochner Laplacian and Bergman kernel expansion of semi-positive line bundles on a Riemann surface, arXiv 1811.00992, (2018).

[3] R. Montgomery, A tour of subriemannian geometries, their geodesics and applications, vol. 91 of Mathematical Surveys and Monographs, American Mathematical Society, Providence, RI, 2002.

[4] L. P. Rothschild and E. M. Stein, Hypoelliptic differential operators and nilpotent groups, Acta Math., 137 (1976), pp. 247–320.

Link (`http://www.mi.uni-koeln.de/~nsavale/teaching.html`)