WS 2017/2018 Dr. Milena Pabiniak

From Calculus to Cohomology

Homework 1

Exercise 1. Using the definitions of *grad*, *rot*, *div* given in the lecture for the case of subsets of $\mathbb{R}^3$, show that

$$rot \circ grad = 0, \qquad div \circ rot = 0.$$

Exercise 2. Prove that if $U \subset \mathbb{R}^3$ is star shaped then $H^2(U) = 0 = H^2(U)$. What is $H^0(U)$ then?
Hint: You may want to play a bit with integrating and with a function $(x_1, x_2, x_3) \mapsto (f_2x_3 - f_3x_2, f_3x_1 - f_1x_3, f_1x_2 - f_2x_1)$ evaluated at tx.

Exercise 3. Decide whether $H^1(\mathbb{R}^3 \setminus S)$ is $\{0\}$ or not, if $S = \{(x_1, x_2, 0);\ x_1^2 + x_2^2 = 1\}$.
Hint: You may want to play a bit with a function $(x_1, x_2, x_3) \mapsto \frac{1}{x_3^2+(x_1^2+x_2^2-1)^2}(-2x_1x_3, -2x_2x_3, x_1^2+x_2^2-1)$.

These exercises are to be discussed on Thursday October 12th.