From Calculus to Cohomology

Homework 10

Exercise 1. Show that rank k vector bundle is trivial if and only if it has k linearly independent sections.

Exercise 2. Assume ξ and η are real vector bundles over B. Pick one out of $\xi \oplus \eta$, $\xi \times_{\mathbb{R}} \eta$, $Hom_{\mathbb{R}}(\xi, \varepsilon^1)$, $Hom_{\mathbb{R}}(\xi, \eta)$, $Alt^k(\xi)$, $F^*(\xi)$ (where $F \colon X \to B$ is some smooth map), and show that this is a vector bundle. Find its rank.

Exercise 3. Suppose that a vector bundle $E \to B = X \times [a, b]$ is trivial when restricted to $X \times [a, c]$ and when restricted to $X \times [c, b]$, for some $b \in (a, b)$. Prove that this vector bundle is trivial.

Exercise 4. Prove Lemma 13.9 from the book. This lemma will be used during the lecture.

These exercises are to be discussed on Thursday, February 1st.