WS 2017/2018 Dr. Milena Pabiniak

From Calculus to Cohomology

Homework 2

Exercise 1. Prove the lemma from the lecture saying that $\omega \in Alt^k(V)$ if and only if $\omega(v_1, \ldots, v_k) = 0$ whenever $v_i = v_{i+1}$, for some $i = 1, \ldots, k-1$.

Exercise 2. Prove that the two definitions of $\omega_1 \wedge \omega_2$ given in the lecture are indeed equivalent. Hint: Remember that a k-linear map is determined by its values on k-tuples $(e_{i_1}, \ldots, e_{i_k})$, where $e_1, \ldots, e_{\dim V}$ is a basis of V.

Exercise 3. Prove that for $\omega_1 \in Alt^k(V)$ and $\omega_2 \in Alt^m(V)$

$$\omega_1 \wedge \omega_2 = (-1)^{km} \omega_2 \wedge \omega_1.$$

Exercise 4. Prove that for $\omega_1 \in Alt^k(V)$, $\omega_2 \in Alt^l(V)$, and $\omega_3 \in Alt^m(V)$

$$(\omega_1 \wedge \omega_2) \wedge \omega_3 = \omega_1 \wedge (\omega_2 \wedge \omega_3).$$

Exercise 5. Find $\omega \in Alt^2(\mathbb{R}^4)$ with $\omega \wedge \omega \neq 0$.

Exercise 6. If $\omega_1, \ldots, \omega_k \in Alt^1(V)$ then for any $v_1, \ldots, v_k \in V$ it holds that

$$\omega_1 \wedge \ldots \wedge \omega_k(v_1, \ldots, v_k) = \det \begin{vmatrix} \omega_1(v_1) & \ldots & \omega_1(v_k) \\ & \ldots & \\ \omega_k(v_1) & \ldots & \omega_k(v_k) \end{vmatrix}$$

These exercises are to be discussed on Thursday October 19th.