WS 2017/2018 Dr. Milena Pabiniak

From Calculus to Cohomology

Homework 6

Exercise 1. Exercises 7.1, 7.4 from the book.

Exercise 2.
Show that the following definitions of a submanifold N in M are equivalent.

i) N is the image of an embedding of a k dimensional manifold.

ii) For all $p \in N$ there exists a chart (U, φ) such that $\varphi(U \cap N) = \varphi(U) \cap (\mathbb{R}^k \times \{0\})$.

Exercise 3.
Prove that if α is a regular value of a smooth map $F\colon M \to N$, such that $L := F^{-1}(\alpha) \neq \emptyset$ then L is a submanifold of M and $T_pL = \ker(dF_p) \subset T_pM$.

Exercise 4. Let $c \in \mathbb{R}$ and $\gamma : \mathbb{R} \to S^1 \times S^1$ be the map

$$\gamma(t) = (e^{2\pi i t}, e^{2\pi i c t}).$$

Is γ injective? immersion? embedding? *Hint: Be careful, your answer should depend on number theoretic properties of c.*

Suggested Exercise 1.
Consider the two atlases $\mathcal{A}_1 = \{(\mathbb{R}, \varphi_1)\}$ and $\mathcal{A}_2 = \{(\mathbb{R}, \varphi_2)\}$ on $\mathbb{R}$ given by $\varphi_1(x) = x$ and $\varphi_2(x) = x^3$.

(a) Show that $\varphi_2^{-1} \circ \varphi_1$ is not differentiable and conclude that the two atlases are not equivalent.

(b) The identity map $\mathrm{id}\colon \{(\mathbb{R}, \varphi_1)\} \to \{(\mathbb{R}, \varphi_2)\}$ is not a diffeomorphism.

(c) The map $f\colon \{(\mathbb{R}, \varphi_1)\} \to \{(\mathbb{R}, \varphi_2)\}$ given by $f(x) = x^3$ is a diffeomorphism. Conclude that the two differentiable structures are diffeomorphic.

These exercises are to be discussed on Thursday November 30th.